



Ageing Europe – An Application of
National Transfer Accounts for Explaining
and Projecting Trends in Public Finances

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1 Introduction

The increasing number of retirees relative to the working-age population will exert fiscal pressure on public transfers programs that provide economic resources to elderly people. The rising concerns about the public dimension of population aging is, however, only one aspect among several being affected by population aging. Indeed, how the private sector (households) reallocates resources from the economically active to the dependent population and its inter-relationship with the public spending are two other important dimensions that population aging will affect. For this reason, the aim of the model developed in the AGENTA project is to simulate not only how population aging affects the reallocation of public resources across generations, but also how households react to changes in demographics and the economy by reallocating privately resources across members.

In order to assess the economic impact of population aging and the consequences of alternative welfare reforms, it is key to understand how individuals respond to changes in demographics, in transfers and in prices. For instance, one should expect that higher life expectancy and educational attainment will lead to an increasing supply of labor by elderly workers and a later entrance in the labor market. Thus, by taking into consideration the behavioral responses, the AGENTA model allows to study how future changes in taxes, social contributions, health care benefits, and other public and private transfers change the use and production of resources within the household. Thus, the strategy followed in the AGENTA project complements many previous works in the field and, especially, the one conducted in the Ageing Working Group for European countries (European Commission, 2015, 2018).

To account for the behavioral reaction, the strategy followed in the AGENTA project has been to set up a model that aims to replicate historical facts like the introduction of the modern educational system as well as from the onset of the demographic transition until now —i.e. from high fertility and high mortality to low fertility and low mortality. As a consequence, projections of future inter-age reallocation of public and private resources are based on the past reactions to changes in the demographic structure. Thus, the strategy followed in the AGENTA project differs from many applied OLG-CGE models, which only replicate the most recent micro- and macroeconomic data.

The economic model developed within the AGENTA project constitutes a large-scale OLG model à la Auerbach and Kotlikoff (1987), which incorporates realistic demography (Bommier and Lee, 2003) and individuals supply their time not only to the market, but also to produce goods and services at home (Sánchez-Romero et al., 2017). Figure 1 illustrates the structure of the model. The model considers three

types of agents (see red circles): households, firms, and a government.

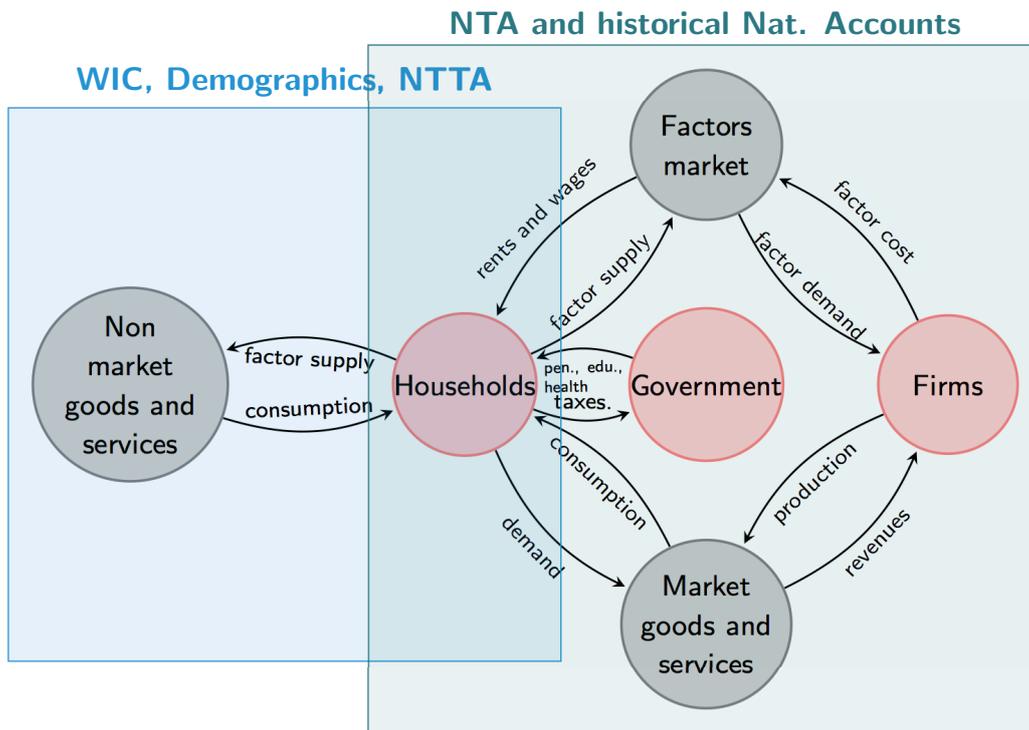


Figure 1: Model framework

The decisions of households are illustrated with arrows pointing to the left-hand side in the case of non-market decisions and to the right-hand side in the case of market decisions. Households consist of an adult, or household head, and a number of dependent children. Individuals are not distinguished by gender (i.e. unisex model) but numbers are consistent with data from population censuses. Individuals face mortality risk à la Yaari (1965) and may live up to a maximum age of 105 years. Adulthood is set at age 16, which implies that when children become 16 years old, they leave their parent’s home and form a new household. The number of dependent children raised varies by age and across cohorts according to the observed and projected fertility and mortality patterns observed in censuses. Household heads are assumed to be heterogeneous by their level of education. We do so by randomly assigning each individual at birth to one of the following three ISCED levels of education developed by UNESCO: ISCED 0-2 (lower secondary education or less), ISCED 3-4 (upper secondary), and ISCED 5+ (tertiary). Hence, each year, individuals differ according to their age, fertility, mortality, and educational attainment.

Household heads endogenously choose the demand for consumption goods, both purchased in the market and produced at home, the supply of labor to the mar-

ket and for the production of home-goods, the time spent on childcare, and the leisure time. Total consumption of market-goods and home-produced goods is distributed among the surviving household members according to their age through the equivalent-adult consumption function frequently used in the National Transfer Accounts (NTA) and the AGENTA project (Istenič et al., 2016). Home-goods are produced combining time and intermediate goods purchased in the market. Moreover, household heads devote time to rear their children, which reduces the available time for work. The time devoted to paid work, unpaid work, and childrearing by the household head is calibrated so as to reproduce the AGENTA data (Istenič et al., 2016). The (net) income generated by household heads is optimally distributed between consumption and savings, which in turn, determines capital accumulation.

Firms combine, as input factors, physical capital and human capital under a constant returns to scale technology to produce a single good that can be either consumed, used as an intermediate good for home-production, or saved as a store of value by individuals. Firms operate in competitive markets, paying for the stock of capital and labor demand their marginal productivities.

The government provides public education to individuals attending school, health care, and other public expenditure, pension benefits to retirees, and distributes all the accidental bequest to surviving individuals. Both public in-kind and in-cash transfers are financed through a balanced PAYG system via consumption taxes, capital income taxes, labor income taxes, and social contributions.

The aim of this deliverable is to give a detailed explanation of the functioning routines of the OLG-CGE model developed in the AGENTA project; that is, how the model works and the assumptions used in its construction. Hence, the document is structured as follows. Section 2 introduces the household problem and the optimal decisions. This section gives an intuitive explanation of the main behavioral reactions of household heads using as an example the profiles derived for three European countries (Austria, France, and Spain).¹ Section 3 details the accumulation of the input factors (physical and human capital) and how the labor-productivity is exogenously calculated. Section 4 explains the underlying assumptions used for projecting public in-kind and in-cash transfers. How the government expenditures are financed through taxes and social contributions is explained in Section 5. All the alternative scenarios run with the model are summarized in Section 6. A list of the main results from a set of alternative simulations is also provided. Section 7 concludes. The equilibrium conditions and the calibration strategy are explained in the Appendix, which also includes an Annex with a summary of the macroeconomic

¹The model has also been applied to Sweden but, thus far, the lack of proper labor income data prevents the inclusion of the results in this report.

implications of the alternative policy reforms listed in Section 6.

2 Households

Time is continuous. Household heads belong to any of the three possible levels of education, which we denote with the letter $e \in \mathcal{E} = \{\text{ISCED 0-2, ISCED 3-4, ISCED 5+}\}$.² Differences across household heads with different level of education are introduced through an educational premium —i.e. the difference in the hourly wage rate between the different educational groups. For expositional clarity and whenever that it is not necessary, we abstract from time, age, and education subscripts in this section.

Household heads derive utility from the consumption of market-produced goods C_m , home-produced goods C_h , childcare time spent per child t_c , and leisure time z . The instantaneous utility U at any age x of a household head is

$$U = \phi_m \log \left(\frac{C_m}{\eta} - \bar{c}_m \right) + \phi_h \log \left(\frac{C_h}{\eta} - \bar{c}_h \right) + \phi_n n \log t_c + \phi_z \frac{z^{1-\sigma} - 1}{1-\sigma}, \quad (1)$$

where η is the total number of equivalent-adult consumers in the household, n is the total number of children measured in terms of childcare time demanded by a new born. For example, a household head that raises a newborn and a four years old child, which requires half of the time of a newborn, then has a n is equal to 1.5 units. $\bar{c}_m, \bar{c}_h > 0$ is the subsistence level of market and home-produced goods, respectively, $\phi_i > 0$ is the relative weight of good $i \in \{m, h, n, z\}$ on the instantaneous utility, and $\sigma > 0$ is the elasticity of substitution on leisure.

Household heads are subject to the following flow budget constraint

$$\dot{k} = \begin{cases} r(1 - \tau_k)k + Bq + (1 - \tau_s - \tau_l)\tilde{w}\ell - (1 + \tau_c)(C_m + C_i) & \text{if } E \leq x \leq R, \\ r(1 - \tau_k)k + Bq + b - (1 + \tau_c)(C_m + C_i) & \text{if } R < x < \omega, \end{cases} \quad (2)$$

where E is the minimum age of entrance in the labor market, R is the age at start claiming retirement benefits, k are the assets held by the household head, \dot{k} denotes the change in assets, r is the market interest rate, $r(1 - \tau_k)k$ is the after-tax capital income, Bq is the accidental bequests distributed by the government to the household head, $(1 - \tau_s - \tau_l)\tilde{w}$ is the (net) wage rate per hour worked, ℓ is the number of hours worked, b is the pension benefit during retirement, and $(1 + \tau_c)(C_m + C_i)$ is the consumption of final and intermediate goods and services purchased in the market. Note that household heads pay direct and indirect taxes,

²Given the lack of historical data, it is assumed that longevity and fertility are uncorrelated to the level of education.

where the set $\{\tau_c, \tau_k, \tau_l, \tau_s\}$ denotes the consumption tax rate, capital income tax rate, labor income tax rate, and social contribution rate, respectively. Individuals are born without assets $k(0) = 0$ and are not allowed to reach the maximum age with debt $k(\omega) \geq 0$.

The wage rate per hour worked of an individual at age x in year t with education e is determined by two components

$$\tilde{w}(x, t, e) = w(t)\varepsilon(x, e), \quad (3)$$

where $w(t)$ is the wage rate per efficient unit of labor and $\varepsilon(x, e)$ is the age-specific productivity by educational attainment.

Home-produced goods and services require the combination of time (t_h) and intermediate goods (C_i). Similar to Greenwood, Seshadri, and Yorukoglu (2005), home-production uses a Cobb-Douglas technology

$$C_h \equiv f(C_i, t_h) = (C_i)^\theta (t_h)^{1-\theta} \text{ with } \theta \in (0, 1), \quad (4)$$

where θ is the intermediate goods share in home-production.

Besides the monetary constraint, individuals are subject to the following time constraint

$$z + t_h + t_c n + \ell \leq T, \quad (5)$$

where T is the net time after subtracting the time devoted to education, which varies depending on the age and the educational attainment. The time devoted to education is exogenously given and taken from the AGENTA database.

2.1 The household problem

Household heads optimally choose at the exact age x the consumption of market goods, home-produced goods, unpaid hours worked, childcare time, and leisure. Let us denote the set of controls $\mathbf{x} = \{C_m, C_i, t_h, t_c, z\}$ and the set of state variables k . The problem is solved using the Hamiltonian

$$H(\mathbf{x}, k, x, \lambda) = S(x)U + \lambda \dot{k}, \quad (6)$$

where $S(x)$ denotes the cohort-specific probability of surviving to age x and λ is the adjacent variable associated to asset holdings. Moreover, household heads are

subject to the Kuhn-Tucker constraints:

$$\zeta_1[T - z - t_h - t_c n - \ell] + \zeta_2 p[f(C_i, t_h) - C_h], \quad (7)$$

where ζ_1 and ζ_2 are the Kuhn-Tucker multipliers.

The first-order conditions from solving (6) subject to (7) are

$$C_m : \quad S(x)\phi_m \frac{1}{C_m - \eta \bar{c}_m} = \lambda(1 + \tau_c), \quad (8)$$

$$C_i : \quad S(x)\phi_h \frac{1}{C_h - \eta \bar{c}_h} \theta \frac{C_h}{C_i} = \lambda(1 + \tau_c), \quad (9)$$

$$t_h : \quad S(x)\phi_h \frac{1}{C_h - \eta \bar{c}_h} (1 - \theta) \frac{C_h}{t_h} = \lambda(1 - \tau_s - \tau_l) \tilde{w} + \zeta_1, \quad (10)$$

$$t_c : \quad S(x)\phi_n \frac{1}{t_c} = \lambda(1 - \tau_s - \tau_l) \tilde{w} + \zeta_1, \quad (11)$$

$$z : \quad S(x)\phi_z \frac{1}{z^\sigma} = \lambda(1 - \tau_s - \tau_l) \tilde{w} + \zeta_1, \quad (12)$$

and the dynamics of the adjacent variable is:

$$\dot{\lambda} = -\lambda r(1 - \tau_k) \Rightarrow \lambda(x) = \lambda(0) e^{-\int_0^x r(t)(1 - \tau_k(t)) dt}. \quad (13)$$

Rearranging terms and assuming an interior solution (i.e., $\zeta_1 = 0$) we can provide the intuition for the first-order conditions by expressing all variables as a function of leisure. Defining the ratio between the minimum consumption of goods and services of type $i \in \{m, h\}$ and the actual consumption of goods and services of type i by $\Delta_i = \frac{\eta \bar{c}_i}{C_i} \in [0, 1)$, we have that the total amount spent buying goods and services in the market is

$$(1 + \tau_c)(C_m + C_i) = (1 - \tau_s - \tau_l) \tilde{w} \left(\frac{\phi_m}{\phi_z} \frac{1}{1 - \Delta_m} + \frac{\theta \phi_h}{\phi_z} \frac{1}{1 - \Delta_h} \right) z^\sigma. \quad (14)$$

Therefore, total market consumption is positively related to leisure (z) and Δ_i for $i \in \{m, h\}$. The total time spent producing goods and services at home is

$$t_h : \quad t_h = \frac{(1 - \theta) \phi_h}{\phi_z} \frac{z^\sigma}{1 - \Delta_h}, \quad (15)$$

and the time per child devoted to childrearing is

$$t_c : \quad t_c = \frac{\phi_n}{\phi_z} z^\sigma. \quad (16)$$

As a consequence, the total time devoted to paid work is

$$\ell : \quad \ell = T - z - \left(\frac{(1 - \theta)\phi_h + n\phi_n(1 - \Delta_h)}{\phi_z(1 - \Delta_h)} \right) z^\sigma. \quad (17)$$

Eqs. (15) and (17) are illustrated in Figure 2. In particular, Figure 2 shows the optimal time devoted to unpaid work (left panel) and paid hours worked (right panel) as a function of leisure time and subsistence level of home-produced goods. Looking at both panels in Figure 2 we can see that unpaid work increases with leisure and the subsistence level, while paid work is negatively related to leisure and the subsistence level. Given that consumption and leisure are both normal goods, an increase in productivity raises consumption and leisure. Hence, higher labor productivity leads to a decrease in the number of paid hours worked and a non-monotonic reaction in unpaid hours (see the red diamonds in Fig. 2(a)). Indeed, unpaid working hours might initially decrease and then increase once that the consumption level is sufficiently higher than the subsistence consumption level (see the left panel in Fig. 2).

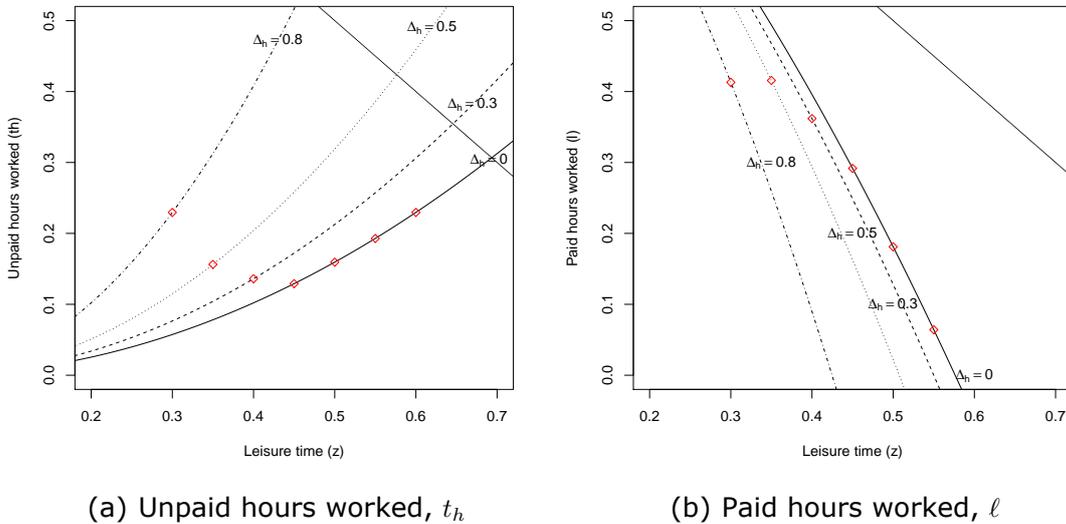


Figure 2: Stylized time devoted to paid and unpaid work as a function of leisure and subsistence level

Analyzing the Frisch elasticity of labor supply, it can be shown that the labor supply of workers becomes more sensitive to changes in the wage rate

$$\frac{\tilde{w}}{\ell} \frac{\partial \ell}{\partial \tilde{w}} = \frac{\tilde{w}}{\ell} \frac{\partial}{\partial \tilde{w}} (T - z - t_h - t_{cn}) = \frac{T - \ell}{\ell} - \left(1 - \frac{1}{\sigma} \right) \frac{z}{\ell} > 0. \quad (18)$$

From (18) we obtain that the Frisch elasticity of labor supply is always positive and increases with leisure. Combining (8) and (12) gives the optimal leisure time

$$z = \left(\frac{\phi_z}{\phi_m} \frac{(1 + \tau_c) C_m}{(1 - \tau_s - \tau_l) \tilde{w}} (1 - \Delta_m) \right)^{\frac{1}{\sigma}}. \quad (19)$$

Thus, the increase in productivity experienced since the industrial revolution, which raises the consumption above the minimum level (i.e. $\Delta_m \uparrow 0$), leads to a progressive increase in leisure and thus, from (18), to a higher elasticity of labor supply. As a consequence, the model captures that individuals are now more sensitive to changes in wages than in the past.

Time use profiles Figure 3 shows the age profiles of paid hours worked (see left panels) and childcare production and childcare consumption (see right panels) in Austria, France, and Spain for three selected years (1900, 2000, and 2100). In the left panels, the age profiles of paid hours worked show three important characteristics. First, the increase in labor-productivity and the educational expansion lead to a later entrance into the labor market. Second, the increase in labor productivity and life expectancy during the working-age from 1900 to 2000 lead to early retirement, while the rise in life expectancy after retirement from 2000 to 2100 leads to an increase in the labor supply at old-age (d’Albis, Sau-Him, and Sánchez-Romero, 2012; Sánchez-Romero, d’Albis, and Prskawetz, 2016).

The panels on the right-hand side of Figure 3 show how each child receives more care time as leisure increases due to the growth of productivity. Despite the increasing time devoted per child, Figure 3 also shows that the time devoted to childcare production decreases due to the fall in the number of children. Therefore, the AGENTA model replicates well the use of time shown in AGENTA database.

From household consumption (C) to individual consumption (c). Household heads optimally decide the consumption of all household members, denoted with the capital letter C . To determine the consumption of each individual living in the household, denoted with the small letter c , equivalent-adult consumption units and the demographics are used. In particular, the model uses the following formula

- Final market goods and services:

$$c_m(x, t) = \begin{cases} \theta(x) \int_x^\omega \frac{m(a-x, t-x) N(a, t)}{N(0, t-x)} \frac{C_m(a, t)}{\eta(a, t)} da & \text{for } 0 \leq x < E, \\ \frac{C_m(x, t)}{\eta(x, t)} & \text{for } x \geq E. \end{cases} \quad (20)$$

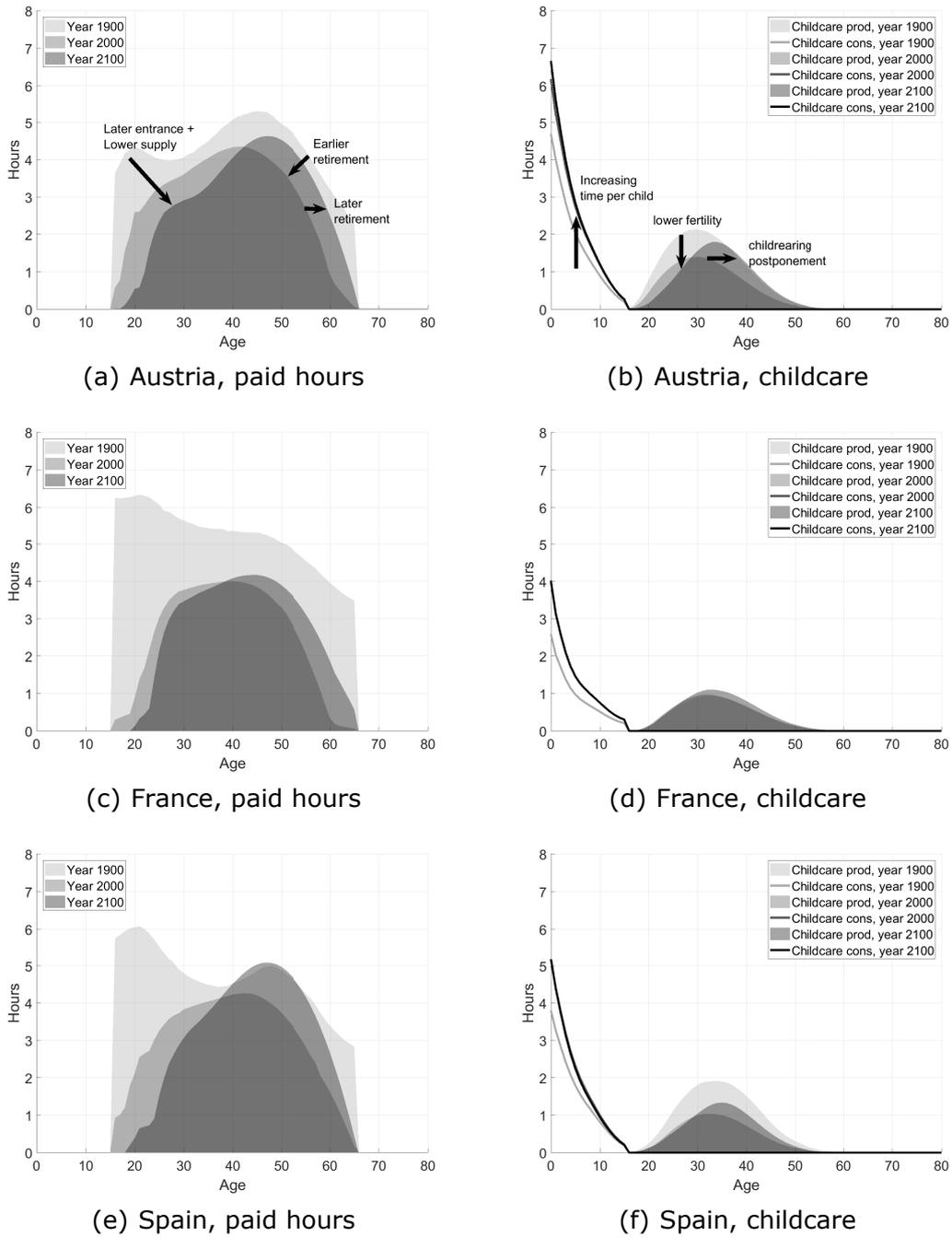


Figure 3: Time-use age profiles in 1900, 2000, and 2100

- Intermediate market goods and services:

$$c_i(x, t) = \begin{cases} \theta(x) \int_x^\omega \frac{m(a-x, t-x)N(a, t)}{N(0, t-x)} \frac{C_i(a, t)}{\eta(a, t)} da & \text{for } 0 \leq x < E, \\ \frac{C_i(x, t)}{\eta(x, t)} & \text{for } x \geq E. \end{cases} \quad (21)$$

- Home-made goods and services:

$$c_h(x, t) = \begin{cases} \theta(x) \int_x^\omega \frac{m(a-x, t-x)N(a, t)}{N(0, t-x)} \frac{C_h(a, t)}{\eta(a, t)} da & \text{for } 0 \leq x < E, \\ \frac{C_h(x, t)}{\eta(x, t)} & \text{for } x \geq E, \end{cases} \quad (22)$$

where $\theta(x)$ is the equivalent-adult consumption at age x , $m(x, t)$ is the fertility rate at age x in year t , $\eta(x, t)$ is the household size measured in terms of equivalent adult consumers, and $N(x, t)$ is the population size of age x in year t . The profile of equivalent-adult consumption units is the standard in NTA, in which $\theta(x)$ equals 0.4 from ages 0 to 4 and rises linearly with age until reaching 1 at the age of 18. The integral term accounts for the expected average consumption of the parent of an individual of age x . The total number of equivalent adult consumers is given by

$$\eta(x, t) = 1 + \int_{x-E}^x \theta(x-a) \frac{S(a, t-x+a)}{S(x, t)} m(a, t-x+a) S(x-a, t-x+a) da, \quad (23)$$

where the first term of the right-hand side accounts for the household head and the integral term accounts for the expected number of surviving children, measured in terms of equivalent adult consumers, born from a parent of age x .

3 Firms

Firms seek to maximise benefits. Benefits are given by the difference between the revenues from the goods and services purchased by households and the cost of paying for the human capital (labor) and physical capital employed in the production process. Firms operate in a perfectly competitive environment and produce one homogenous good according to a standard Cobb-Douglas production function

$$Y(t) = (K(t))^\alpha (A(t)L(t))^{1-\alpha}, \quad (24)$$

where α denotes the share of capital, $K(t)$ is the stock of physical capital, $A(t)$ is labor productivity, and $L(t)$ is the stock of human capital.

The output produced at time t can be either consumed (c_m), used as an intermediary good for home production (c_i), or used as an investment good (\dot{k}) by households (see Eq. (2)). By assuming perfect competition, the rental price of physical capital and human capital equals their marginal products, i.e.

$$r(t) = \alpha \frac{Y(t)}{K(t)} - \delta(t), \quad (25)$$

$$w(t) = (1 - \alpha) \frac{Y(t)}{L(t)}, \quad (26)$$

where $\delta(t)$ is the depreciation rate of physical capital in year t .

The construction of each input factor is detailed below.

Human capital. Workers with different levels of education are assumed to be perfect substitutes. Hence, the stock of human capital is given by

$$L(t) = \int_E^R \int_{\mathcal{E}} \varepsilon(x, e) \ell(x, t, e) N(x, t, e) de dx. \quad (27)$$

where E is the minimum age of entrance in the labor market, R is the maximum age at work, $\varepsilon(x, e)$ is the endowment of efficient labor units at age x with education e , $\ell(x, t, e)$ is the labor supplied by an individual of age x in year t with education e , and $N(x, t, e)$ is the size of the population of age x in year t with education e . The productivity per hour worked and the population size are exogenously given in the model, while the supply of $\ell(x, t, e)$ is optimally chosen by household heads according to Eq. (17).

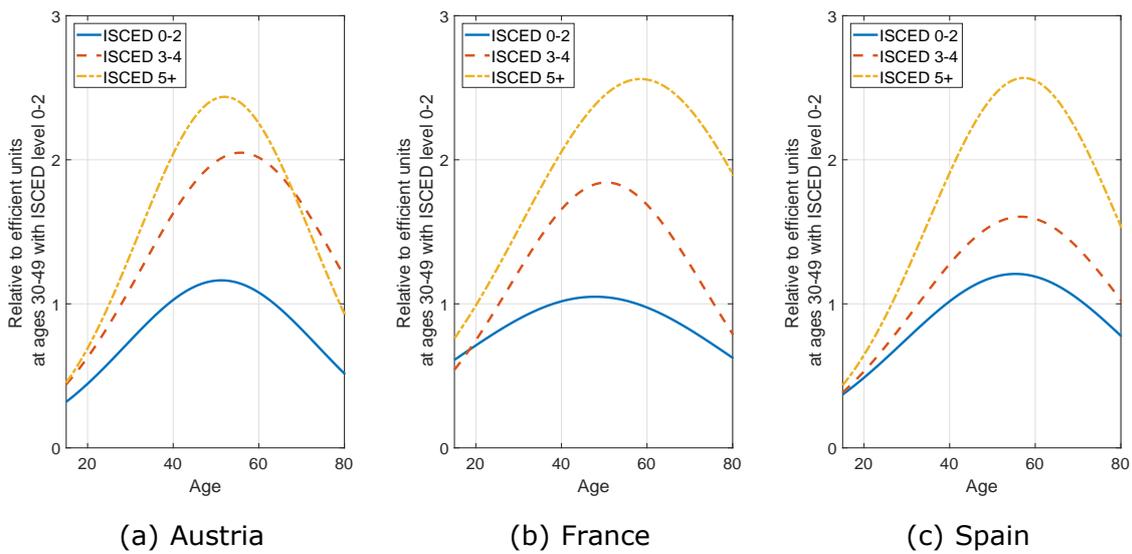


Figure 4: Endowment of efficient labor units by education level.

Source: EU-SILC 2010 for Austria and Spain and from the 1990 Labour Survey for France.

Figure 4 shows the endowment of efficient labor units. The profile of efficient units is calculated deterministically by fitting a quadratic function to the average wage rate per hour worked by educational attainment of men working full-time in the private sector. The information is taken from EU-SILC 2010 for Austria and Spain and from the 1990 Labour Survey for France.

Physical capital. The stock of physical capital is the sum across age and education of the assets held by individuals

$$K(t) = \int_0^\omega \int_{\mathcal{E}} k(x, t, e) N(x, t, e) de dx. \tag{28}$$

Note from Eq. (28) that individuals below age E can hold assets. In particular, assets held at ages below the age of entrance in the labor market (E) come through bequests. For ages older than E , individuals accumulate assets not only because of the received bequests but also because of savings.

Labor productivity. Labor productivity at time t , $A(t)$, is assumed to grow at the exogenous rate of $g_A(t)$, i.e. $\dot{A}(t) = g_A(t)A(t)$. The growth of the labor productivity is exogenously given in the model and calculated applying the formula

$$g_A(t) = \frac{\dot{y}(t)}{y(t)} - \frac{\alpha}{1 - \alpha} \left(\frac{\dot{K}(t)}{K(t)} - \frac{\dot{Y}(t)}{Y(t)} \right) - \left(\frac{\dot{L}(t)}{L(t)} - \frac{\dot{N}(t)}{N(t)} \right), \tag{29}$$

where $y(t)$ is the output per capita in year t and $N(t)$ is the total population size in year t . Numbers for each variable on the right-hand side of Eq (29) are calculated using historical national accounts.

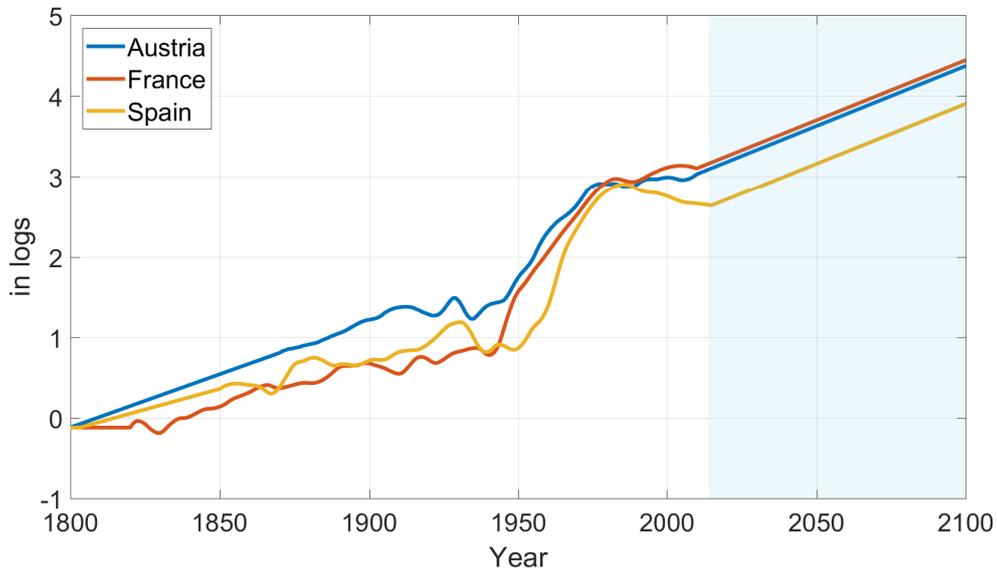


Figure 5: Labor-augmenting technological progress (in logs), period 1800–2100.

Source: Authors’ calculations based on historical national accounts.

Figure 5 shows the reconstructed technological progress used in the benchmark model for Austria, France, and Spain. For the sake of comparison, the capital

share α is assumed to be the same across countries and is held constant over time at 0.275. The value of the capital share is chosen so as to replicate the capital-output ratio observed in each country for the period 1970–2010. The initial labor productivity value in the three countries correspond to the value of 0.886, which is the value derived for France in 1820 based on the assumptions used for the reconstructions. In Austria and Spain, labor productivity grows linearly from the initial value of 0.886 until the first value derived using the country-specific national accounts (1850 in Spain and 1870 in Austria) at a rate equal to the average growth rate observed until War World I. From year 2015 onwards, labor productivity growth is set at 1.5% per year, which is equivalent to a total factor productivity growth of 1.1% per year ($= (1 - 0.275) \times 1.5\%$). Alternative simulations have been run with future annual labor productivity growth rates of 0.5%, 1.0%, and 2.0% (see Section 6).

4 Public expenditures

We assume each government provides in-kind transfers (e.g. education, health care, and others) and in-cash transfers (e.g. retirement benefits).³

4.1 Public in-kind transfers

Public in-kind transfers corresponds in national accounts to public consumption, denoted by G , and is the sum of public expenditures: public expenditures on education, public expenditures on health care, and others public in-kind transfers. Thus, we have

$$G(t) = \text{Pub.Education}(t) + \text{Pub.HealthCare}(t) + \text{Pub.Others}(t), \quad (30)$$

where $\text{Pub.X}(t)$ denotes the total public expenditure on item X as reported in national accounts in year t . Figure 6 shows the total public consumption relative to GDP from 1970 to 2015 and the projected evolution of public consumption relative to GDP for the period 2015–2100 (see shaded blue area). The model suggests an increase in public consumption of around 6 percentage points along the period 2015–2100 in the three countries (Austria, France, and Spain). Next, the underlying assumptions applied for the projection of age-specific public expenditure profiles are detailed.

Educational expenditures. Age-specific profiles of public education expenditures weighted by the average labor income between ages 30–49 are taken from

³Other social benefits such as unemployment benefits, disability benefits, family allowances, parental-leave benefits, and public childcare services are excluded. The cost of the social benefits, other than pension benefits, represent around 5 percent of the GDP in the three countries analyzed.

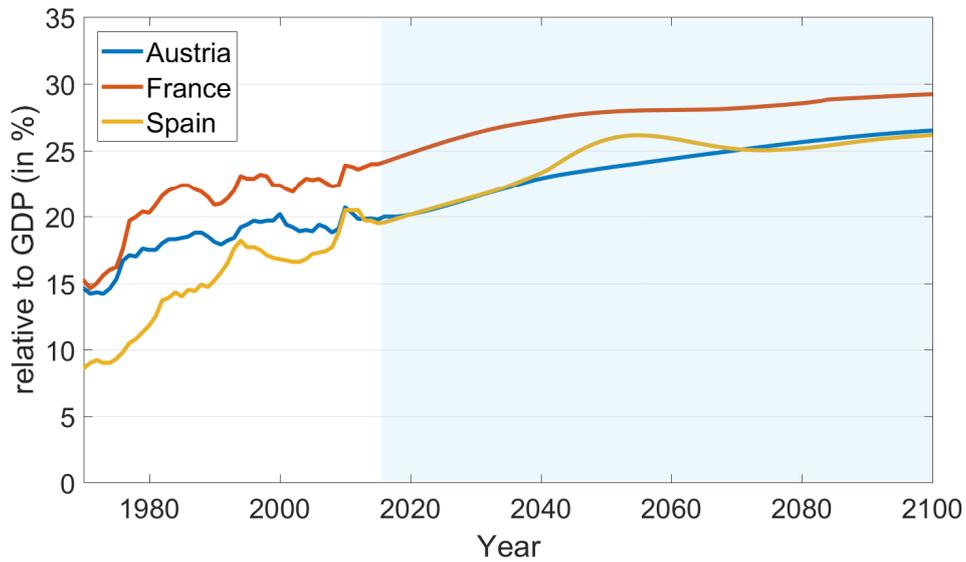


Figure 6: Total public consumption (as % of GDP), period 1970–2100

Source: Authors’ calculations based on historical national accounts.

the AGENTA project, see Istenič et al. (2016), which we denote by $Edu(x, e)$. The age-profiles for Austria, France, and Spain are depicted in Figure 7.

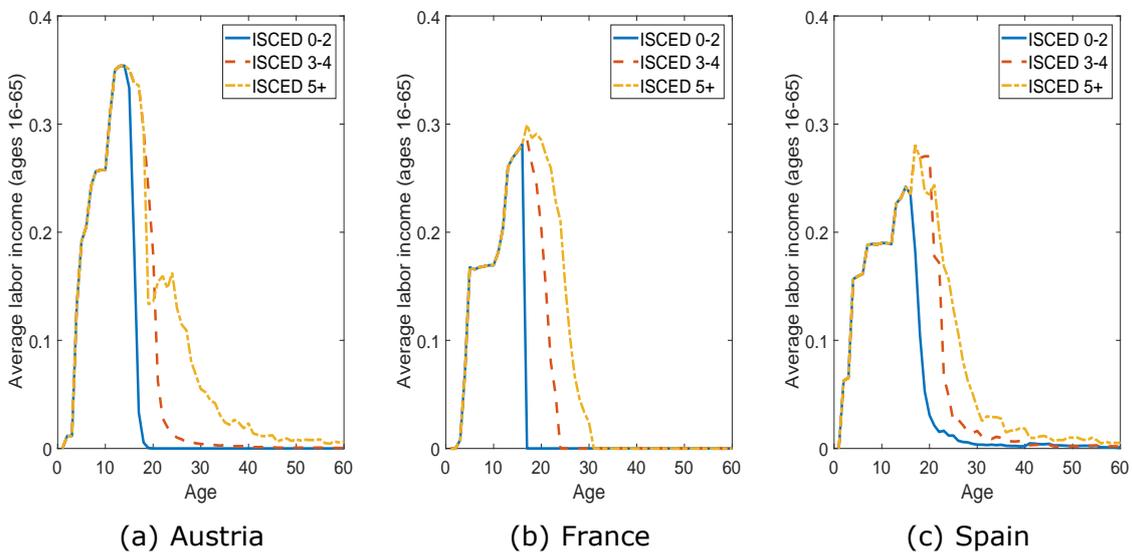


Figure 7: Average expenditure on public education by age (x) and education level (e), $Edu(x, e)$.

Source: Istenič et al. (2016) and authors’ calculations

At each given year t the age-profiles, shown in Fig. 7, are multiplied by the population by level of education $N(x, t, e)$ and by the average labor income for the working-

age population, hereinafter $\bar{w}(t)$. Figure 8 shows the total population by age and by ISCED group (0–2, 3–4, and 5+). Figure 8 is obtained combining the population reconstructed with the educational distribution from the Wittgenstein Centre Database (2015). The projections of the educational attainment in Wittgenstein Centre Database (2015) assume that around 70% of the population born in year 2100 will attain ISCED 5+ and the remaining group, around 30%, will attain ISCED 3-4. The average labor income of the working-age population is internally calculated within the model, which implies that the level of educational expenditures is not fixed over time and may vary because of changes in the economic and demographic conditions.

To get the total public expenditures as observed in national accounts until year 2015, we adjust the level of $\text{Edu}(x, e)$ using the factor $\psi_e(t)$ so as to satisfy

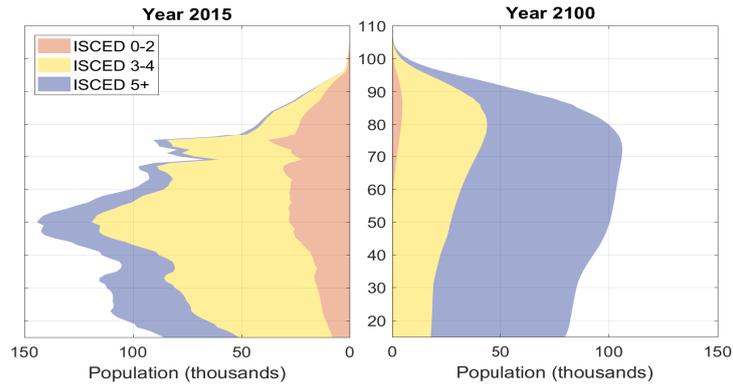
$$\text{Pub.Education}(t) = \psi_e(t)\bar{w}(t) \int_0^\omega \int_E \text{Edu}(x, e)N(x, t, e)dedx, \quad (31)$$

where $\text{Pub.Education}(t)$ is the total public educational expenditure reported in national accounts. To project the public educational expenditure profiles from year 2010 onwards we assume $\psi_e(t)$ at the level obtained in 2015. Thus, the evolution of public expenditures on education depends from 2015 onwards on the change in the age structure of the population, on the educational attainment, and on the average labor income. Figure 9 shows the evolution of the total public expenditure on education relative to GDP until year 2010 (see the white area) and the projection until 2100 (see the blue area).

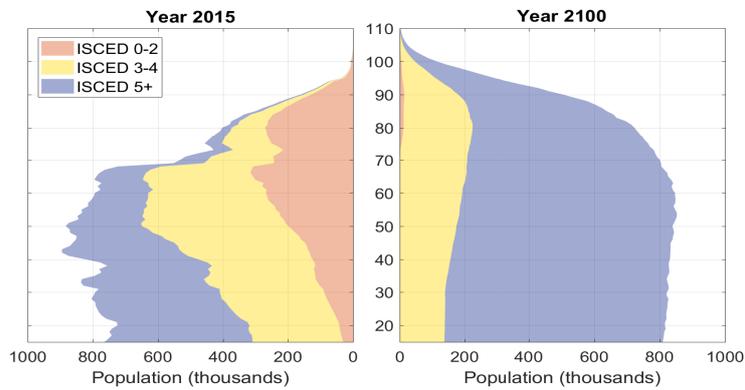
Health care expenditures. Total spending on public health care has more than doubled from 1970 to 2015 according to OECD (2017b). In the coming decades the total health care cost is expected to continue rising because of the ageing of the population. In 2015 the total cost of public health spending relative to the GDP ranged between 6.3% (Spain) and 8.7% (France).

Public health care expenditures are assumed not to differ by education level. Thus, for each country the same age-profile of health care spending is used for the three educational groups. The AGENTA Data Explorer only reports information from age 0 to 80+, hence the per capita cost of health care is assumed to be constant from age 80 onwards as Figure 10 shows.

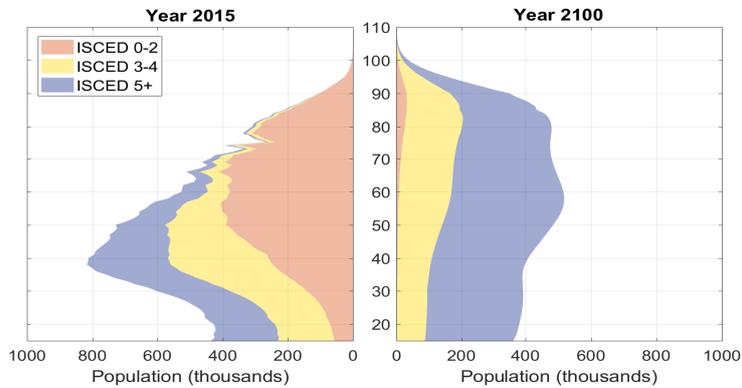
Similar to the educational expenditures, the total expenditure on health care, as reported in national accounts until year 2015, is obtained by adjusting the level of



(a) Austria



(b) France



(c) Spain

Figure 8: Population by age and education in 2015 (left panel) and in 2100 (right panel)

Source: Authors' calculations based on data from Wittgenstein Centre Database (2015)

health(x, e) using the factor $\psi_h(t)$ to satisfy

$$\text{Pub.HealthCare}(t) = \psi_h(t)\bar{w}(t) \int_0^\omega \text{health}(x)N(x, t)dx, \quad (32)$$

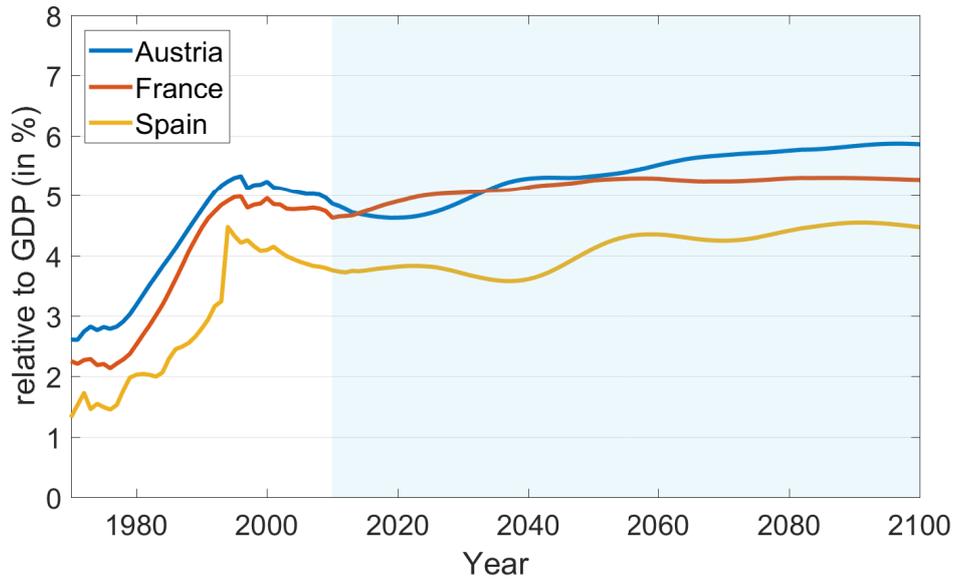


Figure 9: Total public educational expenditure (as % of GDP), period 1970–2100

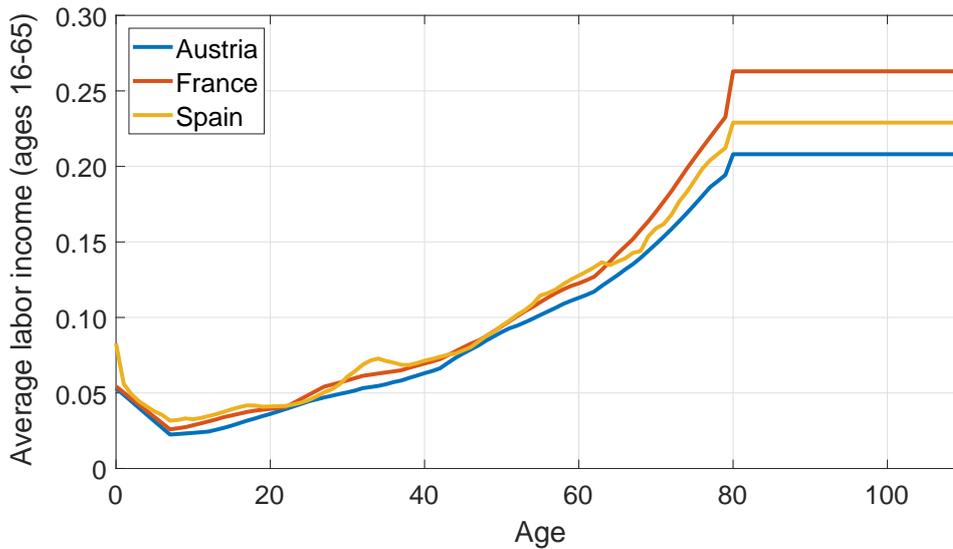


Figure 10: Age profiles of health care costs (relative to the average labor income in the age group 16-65) for Austria, France, and Spain.

Source: Istenič et al. (2017)

where $\bar{w}(t)$ is the average labor income of the working-age population in year t and $\text{Pub.HealthCare}(t)$ is the total public health care expenditure reported in national accounts in year t . From 2015 onwards $\psi_h(t)$ is held constant at the level observed in year 2015. Hence, the projected total health care cost depends on the change in the age structure of the population and on the average labor income.

Figure 11 shows the benchmark results for the total public health care expenditure relative to the GDP from 1970 to 2100 in Austria, France, and Spain. The projections from 2015 onwards (see blue area) suggest that the total health care costs will rise in the three countries. By year 2100, total public health care spending will represent around 12-13% of the GDP. The increase will be more pronounced between 2015 and 2050 especially in Spain.

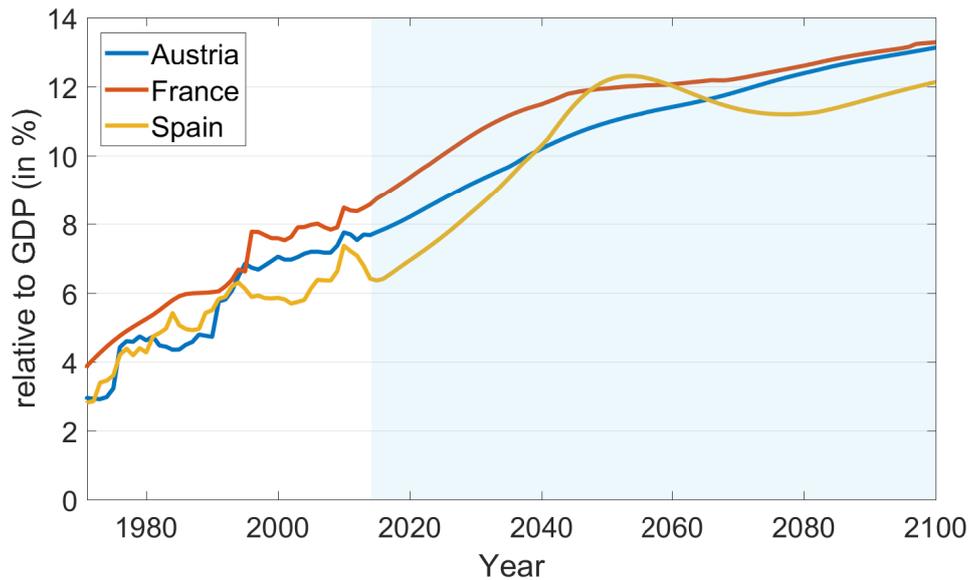


Figure 11: Total public health care expenditure (as % of GDP), period 1970–2100

Other expenditures. Data on total public consumption is available for Austria, France, and Spain from 1920 to 2015, except during World War II. For convenience, other public expenditure is assumed to be the difference between the total public consumption and the sum of the cost of education and health care. Other public consumption is mainly comprised of expenditures on defense, justice and administration. Since this public expenditure is not targeted to specific age groups, other public consumption is equally distributed across the population. From 2015 onwards other public consumption is held constant relative to the GDP. As a consequence, if the sum of public education and health care expenditures continue rising, the total public consumption will also grow.

4.2 Public in-cash transfers

Public pension expenditures. The introduction of pension benefits in the model has a twofold motive. First, public expenditure on pensions is the most important public expenditure program. In 2013, public spending on pensions represented

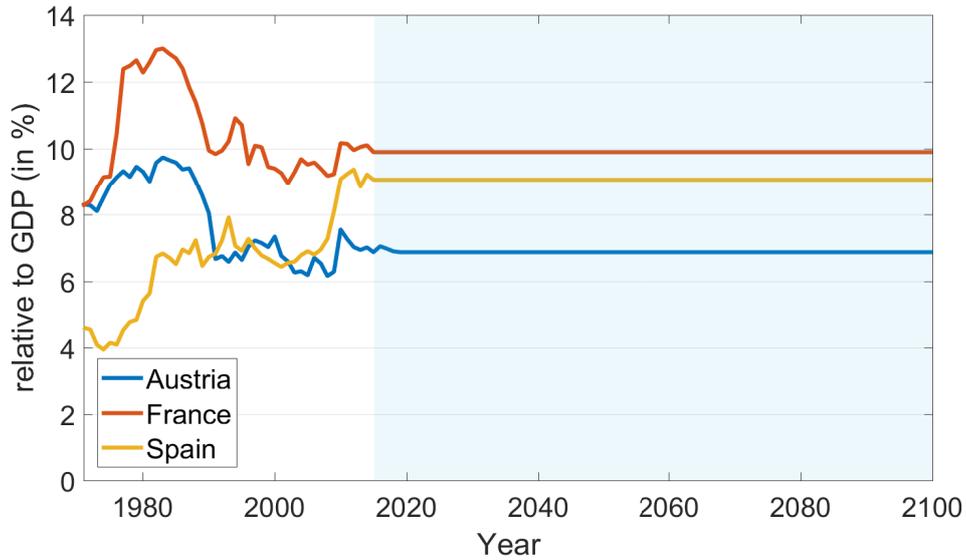


Figure 12: Other public expenditures (as % of GDP), period 1970–2100

around 13.4% of GDP in Austria, 13.8% in France, and 11.4% in Spain. Second, public pension benefits are key for understanding the accumulation of assets, since they influence the saving for retirement motive, which is the main reason for saving.

There are several complexities when trying to model pensions benefits. First, the myriad of pension programs within each country make it unfeasible to efficiently model all population groups. Second, individuals have different labor histories that determine the pension benefit received. And third, public pension profiles are affected by the introduction of several parametric reforms to the system either improving or reducing its generosity. Fortunately, all these differences are reflected in the per capita pension benefit profiles in the NTA database. As a consequence, the model makes use of NTA data by taking the per-capita pension profiles in year 2010. To reconstruct the per capita pension benefit profiles in the past and project them to the future, the same strategy as with the in-kind transfers is followed. In particular, the level of pension benefits is adjusted so as to match historical pension spending, while future profiles of per capita pension benefits can only change because of variations in the average labor income of the working-age population.

The pension expenditure at the exact age x with education e in year t is given by

$$b(x, t, e) = \psi_p(t)\bar{w}(t, e)\text{pen}(x), \tag{33}$$

where $\psi_p(t)$ is average replacement rate in year t , $\bar{w}(t, e)$ is the average labor income of the working-age population with education e in year t , and $\text{pen}(x)$ is the age-shape of the pension benefits observed in the AGENTA database. Multiplying (33) by the

population size and aggregating across all retired individuals, we obtain the total pension expenditures as follows

$$\text{Pub.Pensions}(t) = \int_R^\omega \int_{\mathcal{E}} b(x, t, e) N(x, t, e) de dx, \tag{34}$$

where R sets the age at which individuals start receiving pension benefits and $\text{Pub.Pensions}(t)$ is the total pension benefits claimed in year t .

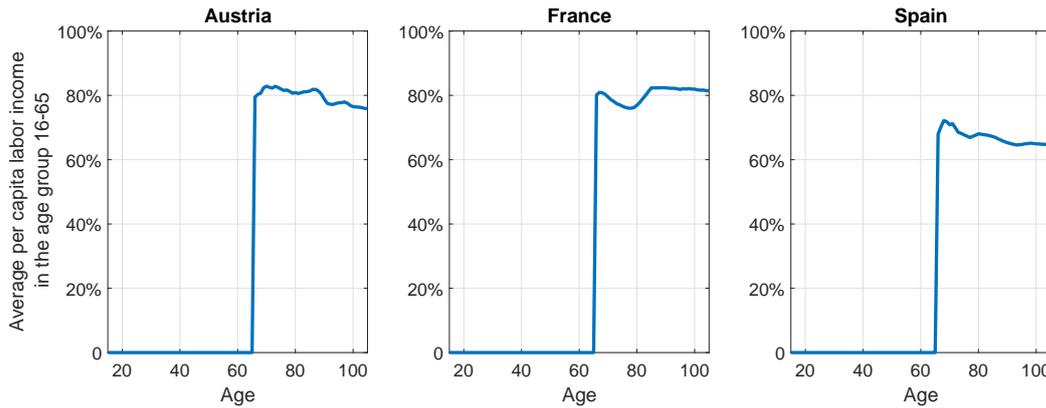


Figure 13: Per capita pension benefit profile, as % of average labor income of the working-age population, in Austria, France, and Spain

Source: Authors’ calculations using data from Istenič et al. (2017). Notes: Profiles have been readjusted upwards so as to obtain the same total pension expenditures to GDP ratio.

Figure 13 shows the average replacement rate relative to average labor income of the working-age population in year 2010. Although according to the OECD (2017a) the pension replacement rate, as a percent of pre-retirement earnings, is 72.3% in Spain, 78.4% in Austria, and 60.5% in France in 2014, the replacement rate relative to the labor income of the working-age population (as given by the European NTA data) is close to 80% in Austria and France and around 70% in Spain. The difference between the values reported using the European NTA database and those from the OECD (2017a) data stems from the fact that the latter assumes full-career workers with average earnings, while the European NTA reflects actual labor income and pension benefits in a given year.

Bequest wealth transfers. The government is assumed to distribute any positive accidental bequests in a lump-sum fashion to the generation containing the decedent’s children.⁴ For those decedents younger than age 28, their positive wealth is assumed to be distributed within the same birth cohort. Any negative wealth left

⁴According to the estimated age-specific fertility rates, the average generational gap is around 28 years.

at death is collected by the government and financed through social contributions. The transmission of wealth transfers are calculated as follows:

First, the government assigns the wealth at death to the probable decedent's children according to

$$\xi(x, t) = \begin{cases} \frac{\mu(x+28, t) \int_{\mathcal{E}} k(x+28, t, e) N(x+28, t, e) de + \mu(x, t) \int_{\mathcal{E}} k(x, t, e) N(x, t, e) de}{N(x, t)} & \text{if } 0 \leq x \leq 28, \\ \frac{\mu(x+28, t) \int_{\mathcal{E}} k(x+28, t, e) N(x+28, t, e) de}{N(x, t)} & \text{if } 28 < x < \omega - 28. \end{cases}$$

If $\xi(x, t)$ is positive, the government transfers the wealth to the surviving children

$$Bq(x, t) = \max\{0, \xi(x, t)\} \text{ (if positive);} \quad (35)$$

If $\xi(x, t)$ is negative, the government collects the debt

$$D_G(t) = \int_0^{\omega-28} \max\{-\xi(x, t), 0\} N(x, t) dx \text{ (if negative),} \quad (36)$$

where $D_G(t)$ is the aggregate debt left at death in year t .

The sum of the positive wealth transferred and the debt left at death in each year t equals all the accidental bequest left

$$\int_0^{\omega} \int_{\mathcal{E}} k(x, t, e) \mu(x, t) N(x, t, e) dedx = \int_0^{\omega-28} Bq(x, t) N(x, t) dx + D_G(t). \quad (37)$$

Note in Eq. (37) that the government distributes the bequest similarly across educational groups, since children may have parents with different education level.

5 Public revenues

Public revenues are included in the model using two main assumptions. First, the model assumes a balanced budget for public in-kind transfers and for public in-cash transfers. Therefore, the government does not issue debt and hence all public expenditures/transfers are financed by taxes and social contributions. Second, in-kind transfers are financed by taxes, while in-cash transfers are financed by social contributions. To allocate the total cost of the in-kind transfers across the set of alternative taxes, we take the tax distribution as observed in historical accounts for tax revenues from Flora (1983), Comín and Díaz (2005), and OECD (2017c). Table 1 reports the average percentage distribution of taxes by major categories

(consumption, capital income, and labor income) over twenty-years periods starting in 1900 for Austria, France, and Spain.

Table 1: Percentage distribution of taxes by major categories (as % of total taxes)

Country	Tax base	Period						
		1900-20	1920-40	1940-60	1960-80	1980-00	2000-15	2015-
Austria	Consumption	0.72	0.73	0.60	0.51	0.49	0.44	0.43
	Capital income	0.21	0.15	0.12	0.13	0.10	0.10	0.10
	Labor income	0.07	0.12	0.28	0.36	0.42	0.46	0.47
France	Consumption	0.78	0.64	0.54	0.44	0.43	0.44	0.47
	Capital income	0.09	0.12	0.19	0.17	0.15	0.17	0.16
	Labor income	0.12	0.23	0.27	0.39	0.41	0.39	0.36
Spain	Consumption	0.57	0.57	0.57	0.49	0.44	0.42	0.44
	Capital income	0.24	0.24	0.24	0.22	0.22	0.25	0.21
	Labor income	0.20	0.20	0.20	0.29	0.35	0.33	0.35

Source: Authors' estimates based on Flora (1983), Comín and Díaz (2005), and OECD (2017c).

By assumption total in-kind public transfers (or public consumption, $G(t)$) is equal to total taxes paid

$$G(t) = T(t), \tag{38}$$

where total taxes are equal to

$$T(t) = \tau_c(t)C(t) + \tau_k(t)r(t)K(t) + \tau_l(t)w(t)L(t). \tag{39}$$

C is the total consumption of goods and services purchased at the market, rK is the total capital income, wL is the total compensation of employees, and $\{\tau_c, \tau_k, \tau_l\}$ is the set of consumption tax rate, capital income tax rate, and labor income tax rate, respectively. Total in-cash public transfers are financed by social contributions. Thus, the social security contribution rate is set at all times so as to finance all pension benefits claimed plus the aggregate debt left at death ($D_G(t)$)

$$\tau_s(t)w(t)L(t) = \int_R^\omega \int_\mathcal{E} b(x, t, e)N(x, t, e)dedx + D_G(t), \tag{40}$$

where $\tau_s(t)$ is the social contribution rate in year t . In order to prevent the cost of pension benefits from causing an excessive burden on future workers, we set the maximum social security contribution rate at 35 percent. In Section 6 the macroeconomic effects of setting a maximum social contribution rate at the level observed in 2015 is presented.

Figure 14 shows in a stacked format the evolution from 1970 to 2100 of the dis-

tribution of public revenues by major categories as a percentage of the GDP. It is important to recall that not all public expenditures are included in the simulations. Still, in year 2015 France has the highest tax revenue (including social contributions) as a percentage to the GDP within the three countries, followed by Austria and Spain. The projection from 2015 to 2100 suggests that tax revenues should progressively increase in the three countries up to levels above 50% of the GDP in order to finance the projected public expenditures.

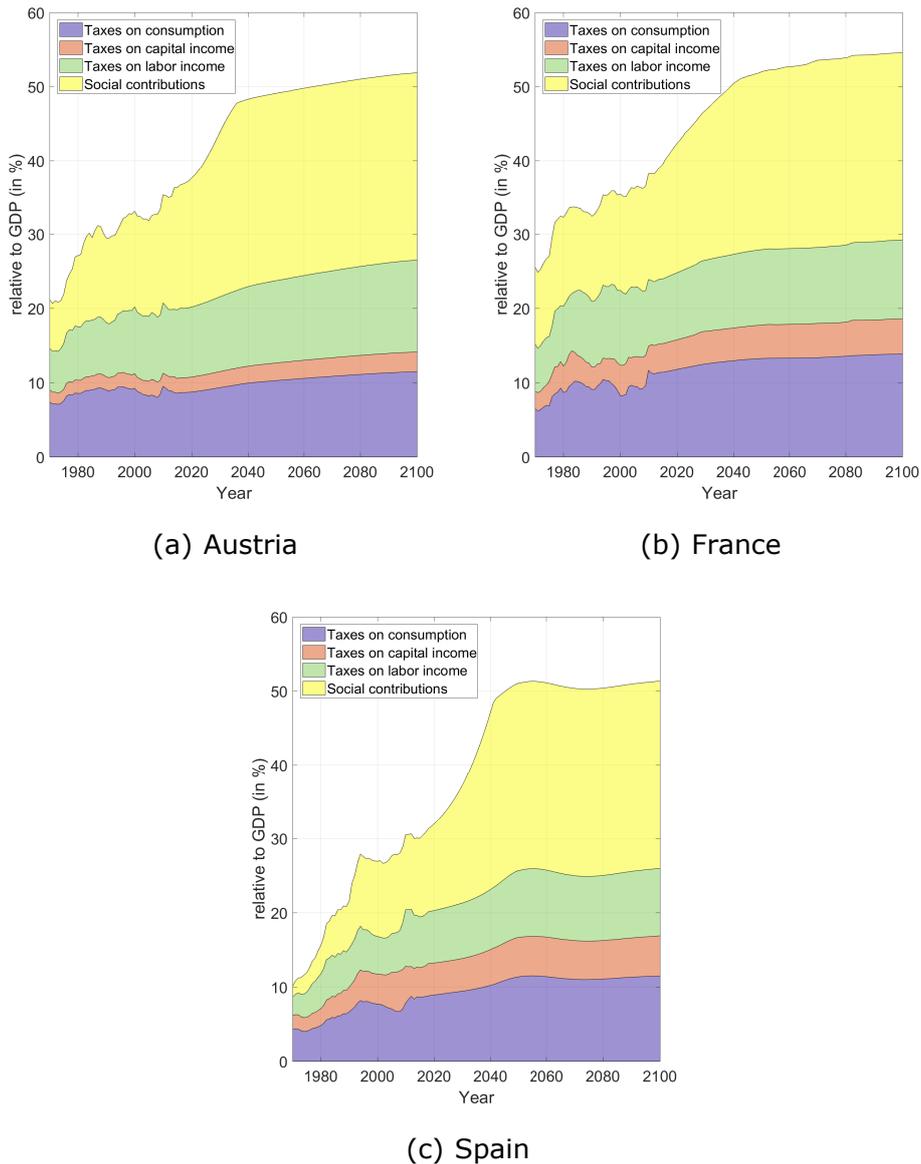


Figure 14: Public revenues by major categories (as % of GDP), period 1970–2100.

Notes: The evolution of social contributions do not consider the implementation of the last pension reforms.

6 Projection scenarios

Table 2 summarizes the set of alternative simulations run using the model. For the sake of space, only the most relevant cases are reported in Annex C. The first set of scenarios (see first row in Table 2) analyzes the macroeconomic impact of different future labor productivity growth rates, ranging from 0.5% to 2%, on output, output per capita, and the input factors (i.e. physical capital and human capital). The second set of scenarios studies the impact of delaying the age at which individuals can start claiming their pension benefits from 65 to 67, 69, and 71. This policy is gradually implemented in year 2020 (phase-in) until 2040 (phase-out). The third set of scenarios analyzes the contribution of the educational expansion on economic growth. This is done by comparing the benchmark scenario, which uses the WIC data, to another simulation in which the educational attainment of future cohorts is kept constant at the level of the cohort born in 1980. The fourth set of scenarios looks at the macroeconomic impact of fully financing the public health care expenditures through a specific tax. We consider three alternative taxes: consumption tax, labour income tax, and capital income tax. For the interpretation of the results it is important to note that we assume that tax rates do not change with wealth, consumption, and income level. The last set of scenarios studies the impact of fixing the social contribution rate at the level observed in 2015. This last scenario implies that the pension system needs to be balanced through the pension replacement rate, which is equivalent to transforming the DB system into a DC system.

Table 2: Summary of alternative simulations

Scenarios	Cases
Labor productivity growth	0.5%, 1.0%, 1.5% , 2.0%
First age at claiming retirement benefits	65 , 67, 69, 71
Educational attainment (WIC Data Explorer)	Medium (SSP2) , Fixed educational attainment from cohort 1980
Health care expenditure	Benchmark , Full Consumption Tax, Full Capital Income Tax, Full Labor Income Tax
Maximum social security contribution	35% , fixed in year 2015

Notes: Bolded cases correspond to the benchmark case scenario.

The main macroeconomic results reported in Annex C are the following:

1st scenario. Our sensitivity analysis shows that the average growth rate of per capita income increases more than proportionally with increases in the growth rate of labor productivity.

2nd scenario. Delaying the age at which individuals start claiming pension benefits slightly increases the total labor input. However, the increase in labor input is partly compensated by a reduction in the accumulation of capital. Thus, the net effect of increasing the age at the start of claiming benefits, for instance, from 65 to 69 results in a relative increase in the growth rate of per-capita income of 0.7% in Austria and Spain and of 8% in France.

3rd scenario. Increasing the proportion of workers with ISCED 5+ from 25% to 75% raises per capita income by 5% in Austria, 7% in France, and has no effect in Spain. This is because individuals with higher education, although they are more productive, enter into the labor market at later ages. Consequently, the total labor input does not significantly change. Moreover, in the case of Spain, workers with higher education retire at the same age, or even earlier, than individuals with lower educational attainment.

This result, however, hints on the necessity of improving the model. In particular, we have not taken into account the fact that some important economic and demographic variables differ by education, such as fertility, mortality, the demand for health care, unemployment, and even the possible complementarity between worker's education and technology.

4th scenario. Fully funding the future public health care expenditures with specific taxes, as opposed to the benchmark case in which the cost is distributed across a set of taxes, shows that the growth rate of per capita income is 5–6% higher with consumption taxes, 3–5% lower with labour income taxes, and 23–25% lower with capital income taxes. Of course, this result is based on the assumption that tax rates are not progressive and that individuals do not link the in-kind transfers received to the taxes paid.

5th scenario. The model results show that the capital-to-output ratio does not significantly increase during the period 2010–2070 (i.e. capital deepening), even when the social contribution rate is fixed at the level of 2015, which the facto reduces the pension replacement rate and increase savings for retirement motive. In this last scenario the growth rate of per-capita income increases 10.5% in Austria, 11.6% in France, and 23.6% in Spain.

7 Conclusions

The final report of Work Package 5 has been prepared for detailing the functioning routines of the overlapping generations-general equilibrium (OLG-CGE) model developed in the AGENTA project. The report contains the underlying assumptions of the model and how they influence on the behavioral reaction of individuals. The report also provides a detailed explanation about how the projections from 2015 to 2100 under the benchmark scenarios has been constructed.

The report is structured according to the main agents represented in the model: households, firms, and a government. The OLG-CGE model has three distinctive features: i) it accounts for the evolution of the population from the beginning of the XIX century, matching historical census data, ii) the model is calibrated so as to replicate historical macroeconomic data, and iii) the model makes use of European NTA and NTTA data. To fully exploit the information collected within the AGENTA project, individuals supply labor not only to the market but also for the production of home-goods. Households are comprised of a household head (an adult older than age 16) and a number of dependent children (younger than age 16), which are consistently reconstructed based on the demographic information collected. Adults endogenously choose the consumption of goods and services, both purchased in the market and produced at home, the supply of labor, the time spent on childcare, and the leisure time. Firms are represented by a neoclassical firm and demand physical capital and human capital. The government provides both in-kind transfers (education, health care, and others) and in-cash transfers (pensions) to each age group based on the information from the European-NTA data.

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A Equilibrium conditions

Let $x \in J = (0, \omega)$, $t \in \mathbf{T} = \{1600, \dots, 2400\}$, and $e \in \mathcal{E}$. Given initial values $\{\bar{c}_m, \bar{c}_h, \phi_{m,e}, \phi_{h,e}, \phi_{n,e}, \phi_{z,e}, \sigma_e, \theta, \alpha, \delta(t), g_A(t), \psi_e(t), \psi_h(t), \psi_p(t), E, R\}_{e \in \mathcal{E}, t \in \mathbf{T}}$, demographics $\{N(x, t, e), \eta(x, t), n(x, t), \mu(x, t)\}_{x \in J, t \in \mathbf{T}, e \in \mathcal{E}}$, and the age-specific productivity endowment by educational attainment $\{\epsilon(x, e)\}_{x \in J, e \in \mathcal{E}}$, a recursive competitive equilibrium is a sequence of a set of household policy functions $\mathbf{x}(x, t, e) \in \mathbf{X}$, government policy functions $\{Bq(x, t), \tau_c(t), \tau_k(t), \tau_l(t), \tau_s(t)\}_{x \in J, t \in \mathbf{T}}$, and factor prices $\{w(t), r(t)\}_{t \in \mathbf{T}}$ such that

1. Factor prices equal their marginal productivities (25)-(26).
2. The government's budget constraints (38)-(40) are satisfied and all accidental bequests equal all transfers given (37).
3. Given the factor prices and government policy functions, household policy functions satisfy eqs. (8)-(13), and the commodity of home production clears.
4. The stock of physical capital and the labor input are given by (27) and (28), respectively.
5. The commodity market clears:

$$Y(t) = C(t) + G(t) + I(t), \quad (41)$$

where the total consumption of market goods is given by

$$C(t) = \int_0^\omega \int_{\mathcal{E}} (c_m(x, t, e) + c_i(x, t, e)) N(x, t, e) dx, \quad (42)$$

and the gross investment in year t is $I(t) = \dot{K}(t) + \delta(t)K(t)$.

B Calibration

The parameters governing the preferences between market-produced goods (ϕ_m), home-produced goods (ϕ_h), childrearing time (ϕ_n), leisure (ϕ_z), and the elasticity of substitution on leisure ($1/\sigma$), for each educational level, are calibrated by minimizing a penalty function (i.e., the sum of the squared errors) that contains information for four time series: per capita income (Y/N), per capita final consumption ($(C+G)/N$), capital-output ratio (K/Y), and average hours worked by each educational group ($\ell(x, e)$).

To reduce the dimension of the set of parameters the following strategy has been followed. First, the sum of the utility weights, excluding that influencing childrearing

time, is assumed to add up to one for all educational groups

$$\phi_m + \phi_h + \phi_z = 1. \quad (43)$$

Second, home-production technology, θ , is set at 0.30 as in Greenwood, Seshadri, and Yorukoglu (2005). Third, using NTTA data we know that home production takes after retirement around 5 hours ($t_h^* = 5/16$) per day while the remaining time of leisure is 11 hours ($z^* = 11/16$). Thus, assuming that the NTTA profiles are not highly affected by the subsistence level in 2010, the utility weight of leisure is

$$\phi_z = (1 - \theta)\phi_h \frac{(z^*)^\sigma}{t_h^*}. \quad (44)$$

Substituting (44) in (43) and rearranging terms we can derive ϕ_h as a function of ϕ_m and σ according to

$$\phi_h = \frac{1 - \phi_m}{1 + (1 - \theta) \frac{(z^*)^\sigma}{t_h^*}}. \quad (45)$$

From (11) and (12) the time spent on a newborn child is equal to $t_c = \frac{\phi_n}{\phi_z} z^\sigma$. Thus, using NTTA data we have

$$\phi_n = \phi_z \frac{t_c}{z^\sigma}. \quad (46)$$

It is important realize that, ceteris paribus other parameters, a high value of σ implies an overall reduction in the labor supply, mainly at prime working ages, and also a shift in the time preference between early and late labor supply.

All parameter values are introduced in Table 3.

Table 3: Benchmark model parameters

General parameters	Symbol	Values			
<i>Firms technology</i>					
Capital share	α	0.275			
Capital depreciation rate	$\delta(t)$	National accounts			
Labor-augmenting technology	$A(t)$	Authors' estimates			
<i>Home production</i>					
Intermediate goods share	θ	0.300			
<i>Household preferences</i>					
Age at parental leave	E	16			
Retirement age	R	65			
Country-specific parameters	Symbol	Values			
Austria					
Subsistence level market goods	\bar{c}_m	0.261			
Subsistence level home goods	\bar{c}_m	0.015			
Level of education					
			Primary or less	Secondary	Tertiary
IES on leisure	σ^e	1.912	2.438	2.732	
Weight of market goods	ϕ_m^e	0.196	0.157	0.176	
Weight of home goods	ϕ_h^e	0.384	0.444	0.456	
Weight of childcare	ϕ_n^e	0.510	0.714	0.816	
Weight of leisure	ϕ_z^e	0.420	0.399	0.367	
France					
Subsistence level market goods	\bar{c}_m	0.398			
Subsistence level home goods	\bar{c}_m	0.030			
Level of education					
			Primary or less	Secondary	Tertiary
IES on leisure	σ^e	1.523	2.081	1.962	
Weight of market goods	ϕ_m^e	0.080	0.102	0.108	
Weight of home goods	ϕ_h^e	0.406	0.443	0.430	
Weight of childcare	ϕ_n^e	0.325	0.467	0.428	
Weight of leisure	ϕ_z^e	0.514	0.455	0.462	
Spain					
Subsistence level market goods	\bar{c}_m	0.300			
Subsistence level home goods	\bar{c}_m	0.020			
Level of education					
			Primary or less	Secondary	Tertiary
IES on leisure	σ^e	1.750	1.950	1.150	
Weight of market goods	ϕ_m^e	0.100	0.120	0.100	
Weight of home goods	ϕ_h^e	0.416	0.423	0.366	
Weight of childcare	ϕ_n^e	0.505	0.543	0.377	
Weight of leisure	ϕ_z^e	0.484	0.457	0.534	

C Annex

Table 4: Austria: Benchmark projection

	2010-19	2020-29	2030-39	2040-49	2050-59	2060-69
<i>Demographics</i>						
Fertility	1.45	1.48	1.52	1.56	1.59	1.62
Life expectancy						
at birth	80.3	81.5	82.8	84.1	85.3	86.4
at 65	19.7	19.9	20.9	21.8	22.7	23.6
Population (millions)	8.30	8.68	9.14	9.48	9.61	9.55
0-15 as % total population	16.2	15.2	15.2	15.0	14.7	14.7
16-69 as % total population	72.1	70.9	68.9	65.2	64.3	63.5
70+ as % total population	11.7	13.9	15.8	19.8	21.0	21.8
<i>Macroeconomics</i>						
Output (growth rate)	2.39	1.64	1.36	1.87	1.68	1.71
Labor input (growth rate)	0.97	-0.01	0.09	0.52	0.17	0.19
Labor productivity (growth rate)	1.67	1.49	1.49	1.49	1.49	1.49
Per capita output (growth rate)	1.96	1.12	0.97	1.70	1.73	1.81
Capital-to-output (growth rate)	2.49	2.33	2.43	2.29	2.21	2.22

Table 5: Austria: Alternative projection scenarios (in percentage)

	2010-19	2020-29	2030-29	2040-49	2050-59	2060-69
<i>Labor productivity 1,00%</i>						
Output (growth rate)	1,91	0,91	1,10	1,25	1,04	1,13
Labor input (growth rate)	0,81	-0,29	0,39	0,32	0,04	0,10
Labor productivity (growth rate)	1,17	1,00	1,00	1,00	1,00	1,00
Per capita output (growth rate)	1,47	0,39	0,73	1,11	1,10	1,22
Capital-to-output (growth rate)	-0,20	0,56	-0,79	-0,20	0,15	0,00
<i>Labor productivity 2,00%</i>						
Output (growth rate)	2,99	1,83	2,04	2,20	2,05	2,11
Labor input (growth rate)	1,20	-0,31	0,36	0,28	0,04	0,10
Labor productivity (growth rate)	2,06	1,98	1,98	1,98	1,98	1,98
Per capita output (growth rate)	2,55	1,31	1,68	2,06	2,11	2,20
Capital-to-output (growth rate)	-1,37	0,54	-0,78	-0,11	0,04	0,05
<i>Start pension claims 67</i>						
Output (growth rate)	2,34	1,60	1,42	1,72	1,56	1,62
Labor input (growth rate)	0,96	-0,03	0,11	0,37	0,05	0,11
Labor productivity (growth rate)	1,61	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	1,90	1,09	1,06	1,58	1,63	1,71
Capital-to-output (growth rate)	-0,58	0,41	-0,50	-0,34	0,05	0,05
<i>Start pension claims 69</i>						
Output (growth rate)	2,29	1,79	1,66	1,39	1,56	1,62
Labor input (growth rate)	0,91	0,22	0,26	0,00	0,09	0,12
Labor productivity (growth rate)	1,61	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	1,85	1,27	1,29	1,24	1,62	1,72
Capital-to-output (growth rate)	-0,59	0,23	-0,26	-0,27	-0,06	0,04
<i>Fixed educational attainment</i>						
Output (growth rate)	2,61	1,26	1,49	1,56	1,38	1,44
Labor input (growth rate)	1,11	-0,43	0,29	0,12	-0,15	-0,08
Labor productivity (growth rate)	1,61	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	2,17	0,75	1,12	1,42	1,45	1,53
Capital-to-output (growth rate)	-0,98	0,64	-0,78	-0,07	0,07	0,04
<i>Health: Consumption tax</i>						
Output (growth rate)	2,28	1,56	1,89	1,78	1,57	1,61
Labor input (growth rate)	0,93	-0,06	0,55	0,24	0,02	0,09
Labor productivity (growth rate)	1,61	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	1,84	1,04	1,52	1,64	1,63	1,71
Capital-to-output (growth rate)	-0,67	0,37	-0,43	0,16	0,15	0,08
<i>Health: Labor income tax</i>						
Output (growth rate)	2,59	1,28	1,19	1,67	1,56	1,62
Labor input (growth rate)	1,16	-0,57	-0,02	0,35	0,07	0,11
Labor productivity (growth rate)	1,61	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	2,15	0,77	0,83	1,53	1,62	1,72
Capital-to-output (growth rate)	-0,46	0,98	-0,77	-0,40	-0,01	0,06
<i>Health: Capital income tax</i>						
Output (growth rate)	2,04	0,87	0,97	1,30	1,42	1,58
Labor input (growth rate)	0,71	-0,24	0,89	0,50	0,05	0,06
Labor productivity (growth rate)	1,61	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	1,60	0,35	0,61	1,16	1,48	1,67
Capital-to-output (growth rate)	-0,75	-1,02	-3,74	-1,75	-0,34	0,09
<i>Social contribution rate at 2015 level</i>						
Output (growth rate)	2,41	2,06	1,85	1,73	1,54	1,58
Labor input (growth rate)	1,05	0,55	0,41	0,23	0,00	0,06
Labor productivity (growth rate)	1,61	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	1,97	1,54	1,49	1,58	1,61	1,67
Capital-to-output (growth rate)	-0,76	0,05	-0,05	0,00	0,14	0,07

Table 6: France: Benchmark projection

	2010-19	2020-29	2030-39	2040-49	2050-59	2060-69
<i>Demographics</i>						
Fertility	2,00	2,02	2,00	2,00	1,99	1,98
Life expectancy						
at birth	81,4	83,3	84,7	86,0	87,1	88,1
at 65	20,5	21,8	22,6	23,5	24,3	25,1
Population (millions)	62,32	65,13	67,69	69,88	71,33	72,40
0-15 as % total population	19,6	19,2	18,3	18,1	18,1	17,8
16-69 as % total population	67,9	66,5	64,3	62,2	61,3	61,3
70+ as % total population	12,5	14,3	17,3	19,7	20,5	20,9
<i>Macroeconomics</i>						
Output (growth rate)	1,40	1,64	1,45	1,51	1,93	1,76
Labor input (growth rate)	0,39	0,34	0,06	0,24	0,44	0,25
Labor productivity (growth rate)	1,23	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	0,96	1,25	1,14	1,31	1,78	1,60
Capital-to-output (growth rate)	-0,57	-0,52	-0,31	-0,17	-0,40	0,06

Table 7: France: Alternative projection scenarios (in percentage)

	2010-19	2020-29	2030-29	2040-49	2050-59	2060-69
<i>Labor productivity 1,00%</i>						
Output (growth rate)	1,03	1,18	0,95	1,27	1,17	1,31
Labor input (growth rate)	0,30	0,34	0,06	0,45	0,27	0,23
Labor productivity (growth rate)	0,78	1,00	1,00	1,00	1,00	1,00
Per capita output (growth rate)	0,59	0,79	0,63	1,06	1,02	1,15
Capital-to-output (growth rate)	-0,12	-0,44	-0,31	-0,53	0,17	-0,14
<i>Labor productivity 2,00%</i>						
Output (growth rate)	1,77	2,13	1,70	2,32	2,29	2,22
Labor input (growth rate)	0,48	0,36	-0,13	0,45	0,36	0,22
Labor productivity (growth rate)	1,67	1,98	1,98	1,98	1,98	1,98
Per capita output (growth rate)	1,33	1,75	1,38	2,12	2,14	2,07
Capital-to-output (growth rate)	-1,01	-0,61	0,07	-0,78	-0,11	0,14
<i>Start pension claims 67</i>						
Output (growth rate)	1,34	1,79	1,71	1,65	1,72	1,87
Labor input (growth rate)	0,33	0,52	0,34	0,30	0,24	0,35
Labor productivity (growth rate)	1,23	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	0,90	1,40	1,39	1,45	1,57	1,71
Capital-to-output (growth rate)	-0,57	-0,62	-0,33	-0,20	-0,11	-0,03
<i>Start pension claims 69</i>						
Output (growth rate)	1,29	1,93	1,90	1,62	1,77	1,84
Labor input (growth rate)	0,28	0,71	0,52	0,19	0,30	0,35
Labor productivity (growth rate)	1,23	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	0,85	1,54	1,58	1,42	1,62	1,69
Capital-to-output (growth rate)	-0,56	-0,73	-0,29	-0,13	-0,06	-0,01
<i>Fixed educational attainment</i>						
Output (growth rate)	1,41	1,66	1,08	1,53	1,76	1,66
Labor input (growth rate)	0,40	0,33	-0,31	0,14	0,38	0,15
Labor productivity (growth rate)	1,23	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	0,97	1,28	0,76	1,33	1,61	1,50
Capital-to-output (growth rate)	-0,57	-0,47	0,22	-0,73	-0,30	0,19
<i>Health: Consumption tax</i>						
Output (growth rate)	1,30	1,83	1,77	1,75	1,80	1,77
Labor input (growth rate)	0,32	0,56	0,20	0,28	0,32	0,23
Labor productivity (growth rate)	1,23	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	0,86	1,45	1,46	1,54	1,65	1,61
Capital-to-output (growth rate)	-0,63	-0,56	0,20	-0,04	-0,01	0,07
<i>Health: Labor income tax</i>						
Output (growth rate)	1,73	1,61	0,85	1,68	1,80	1,70
Labor input (growth rate)	0,66	0,05	-0,63	0,52	0,40	0,17
Labor productivity (growth rate)	1,23	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	1,29	1,22	0,54	1,48	1,65	1,54
Capital-to-output (growth rate)	-0,42	0,21	-0,01	-0,93	-0,18	0,13
<i>Health: Capital income tax</i>						
Output (growth rate)	0,98	0,99	0,93	1,33	1,61	1,80
Labor input (growth rate)	0,03	0,31	0,65	0,52	0,36	0,34
Labor productivity (growth rate)	1,23	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	0,54	0,60	0,61	1,12	1,46	1,65
Capital-to-output (growth rate)	-0,74	-2,14	-3,20	-1,78	-0,54	-0,26
<i>Social contribution rate at 2015 level</i>						
Output (growth rate)	1,28	1,94	1,87	1,78	1,90	1,86
Labor input (growth rate)	0,26	0,72	0,49	0,35	0,39	0,32
Labor productivity (growth rate)	1,23	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	0,84	1,55	1,55	1,58	1,75	1,70
Capital-to-output (growth rate)	-0,47	-0,86	-0,29	0,00	-0,03	0,12

Table 8: Spain: Benchmark projection

	2010-19	2020-29	2030-39	2040-49	2050-59	2060-69
<i>Demographics</i>						
Fertility	1,42	1,36	1,41	1,46	1,50	1,54
Life expectancy						
at birth	81,7	82,8	84,0	85,1	86,2	87,2
at 65	20,3	20,8	21,7	22,5	23,4	24,1
Population (millions)	46,19	45,91	44,36	43,99	44,42	44,36
0-15 as % total population	15,9	15,6	12,6	11,9	13,1	13,3
16-69 as % total population	71,8	70,0	69,8	65,4	59,1	58,7
70+ as % total population	12,3	14,4	17,6	22,7	27,8	28,1
<i>Macroeconomics</i>						
Output (growth rate)	0,96	1,60	0,06	1,55	1,79	1,76
Labor input (growth rate)	1,23	0,27	-1,68	0,78	0,59	0,20
Labor productivity (growth rate)	0,33	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	1,02	1,94	0,14	1,45	1,80	1,90
Capital-to-output (growth rate)	-1,58	-0,18	0,55	-2,06	-0,64	0,13

Table 9: Spain: Alternative projection scenarios (in percentage)

	2010–19	2020–29	2030–29	2040–49	2050–59	2060–69
<i>Labor productivity 1,00%</i>						
Output (growth rate)	0,88	1,22	-0,40	1,00	1,33	1,28
Labor input (growth rate)	1,23	0,28	-1,64	0,73	0,65	0,23
Labor productivity (growth rate)	0,13	1,00	1,00	1,00	1,00	1,00
Per capita output (growth rate)	0,95	1,56	-0,31	0,90	1,35	1,42
Capital-to-output (growth rate)	-1,25	-0,10	0,65	-1,98	-0,76	0,12
<i>Labor productivity 2,00%</i>						
Output (growth rate)	1,03	2,09	0,58	1,89	2,30	2,25
Labor input (growth rate)	1,22	0,33	-1,61	0,63	0,58	0,20
Labor productivity (growth rate)	0,52	1,98	1,98	1,98	1,98	1,98
Per capita output (growth rate)	1,09	2,44	0,67	1,80	2,32	2,39
Capital-to-output (growth rate)	-1,90	-0,55	0,57	-1,93	-0,67	0,18
<i>Start pension claims 67</i>						
Output (growth rate)	0,91	1,79	0,57	0,94	1,76	1,80
Labor input (growth rate)	1,18	0,48	-1,07	-0,01	0,62	0,25
Labor productivity (growth rate)	0,33	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	0,97	2,14	0,65	0,85	1,78	1,94
Capital-to-output (growth rate)	-1,60	-0,45	0,54	-1,61	-0,80	0,05
<i>Start pension claims 69</i>						
Output (growth rate)	0,86	1,85	1,11	0,39	1,85	1,73
Labor input (growth rate)	1,13	0,59	-0,48	-0,82	0,81	0,22
Labor productivity (growth rate)	0,33	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	0,92	2,19	1,19	0,29	1,86	1,87
Capital-to-output (growth rate)	-1,60	-0,60	0,28	-0,79	-1,25	0,12
<i>Fixed educational attainment</i>						
Output (growth rate)	0,97	1,63	0,01	1,56	1,79	1,76
Labor input (growth rate)	1,23	0,30	-1,72	0,79	0,58	0,21
Labor productivity (growth rate)	0,33	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	1,03	1,98	0,09	1,46	1,80	1,91
Capital-to-output (growth rate)	-1,59	-0,21	0,59	-2,06	-0,64	0,13
<i>Health: Consumption tax</i>						
Output (growth rate)	0,92	1,84	0,41	1,58	1,85	1,70
Labor input (growth rate)	1,20	0,49	-1,42	0,57	0,55	0,16
Labor productivity (growth rate)	0,33	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	0,98	2,18	0,50	1,48	1,87	1,85
Capital-to-output (growth rate)	-1,67	-0,29	0,98	-1,39	-0,46	0,18
<i>Health: Labor income tax</i>						
Output (growth rate)	1,05	1,84	-0,61	1,40	1,86	1,79
Labor input (growth rate)	1,28	0,24	-2,55	0,77	0,75	0,27
Labor productivity (growth rate)	0,33	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	1,11	2,18	-0,52	1,31	1,87	1,93
Capital-to-output (growth rate)	-1,16	0,12	1,32	-2,39	-0,99	0,10
<i>Health: Capital income tax</i>						
Output (growth rate)	0,63	1,24	-0,16	0,98	1,51	1,62
Labor input (growth rate)	0,97	0,32	-0,99	0,76	0,64	0,21
Labor productivity (growth rate)	0,33	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	0,70	1,59	-0,08	0,88	1,52	1,76
Capital-to-output (growth rate)	-1,72	-1,59	-1,58	-3,38	-1,68	-0,19
<i>Social contribution rate at 2015 level</i>						
Output (growth rate)	0,99	2,10	1,67	1,20	1,80	1,89
Labor input (growth rate)	1,32	0,86	0,10	-0,13	0,54	0,34
Labor productivity (growth rate)	0,33	1,49	1,49	1,49	1,49	1,49
Per capita output (growth rate)	1,05	2,45	1,75	1,11	1,82	2,03
Capital-to-output (growth rate)	-1,76	-0,62	0,06	-0,21	-0,67	0,16