Ageing Europe – An Application of National Transfer Accounts for Explaining and Projecting Trends in Public Finances

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DELIVERABLE 5.1 Retrospective calibration of overlapping generations general equilibrium models (OLG-CGE)

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Authors: Gemma Abio, abio@ub.edu

Concepció Patxot, cio.patxot@ub.edu

Elisenda Rentería, elisenda.renteria@gmail.com

Miguel Sánchez-Romero, miguel.sanchez@oeaw.ac.at

Guadalupe Souto, guadalupe.souto@uab.es

Coordinator: Alexia Fürnkranz-Prskawetz

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Contents

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1 Introduction

The Work Package 5 (WP5) develops a simulation model based on NTA and NTTA data in order to evaluate the impact of ageing on the sustainability of public finances.

This report has a twofold motive. First, to request retrospective data to the countries involved in the AGENTA project for the historical reconstruction of their populations, the historical reconstruction of productivity, and to reconstruct the past evolution of public revenues and expenditures. Second, to explain the underlying models we will implement for the reconstruction of the population and productivity, which later on will be used as an input for the OLG-CGE model.

The development of NTA/NTTA -based overlapping generations (OLG) - computable general equilibrium (CGE) models provides a powerful tool for both academic and policy-oriented analysis of societies where the age of individuals, the age composition of a population, and the birth year of a cohort all matter. With respect to other dynamic general equilibrium models like the infinite horizon representative agent, the OLG models allow for an explicit account of the relevant life cycle events and the interaction between generations with different demographic characteristics. In our analysis, CGE models will constitute the framework for simulating the role of the demographic structure for the future of taxes and public transfers, taking into account some of the behavioural reactions of individuals.

The NTA data set enables us to produce a full imputation of National Accounts by age, so that most macro magnitudes are expressed in age profiles. Supplemented by a full accounting of net transfers paid to the public coffers by citizens, this implies a full account of private transfers and asset-based reallocations by age. Finally, the inclusion of time use estimates from recent surveys adds time transfers to complete the picture. Overall, this data set will make possible the derivation of intra and intergenerational transfers when reforms proposed for the sustainability of the welfare state are simulated. As explained below, the main challenge is designing a model capable of replicating the main changes associated to the demographic transition and offer at the same time sound projections to the future.

2 The modelling strategy

To better explain the past and more plausibly predict the future of inter-age reallocations including the public tax-transfer system, we will extend OLG-CGE models to incorporate more detailed demographic information and realistic household transfers. This way, the main strength of NTA/NTTA-based OLG-CGE modelling is the combination of sound theoretical underpinning with a rich data set. This combination will allow us to calibrate the model involving several decision processes and contrast the simulation results with historical patterns of consumption and savings together with labour market decisions, educational investment and non-market work observed in reality. In addition, incorporating demographic data (i.e. applying more detailed demographic measures and distinctions) will allow us to develop a procedure for reproducing the structure of households and the transfers occurring among them. Finally, the consideration of private transfers, including time transfers, in combination with public transfers improves the possibilities of obtaining meaningfully inclusive results when simulating reforms.

In order to design the modelling strategy, it is important to keep in mind the various socioeconomic changes associated with the demographic transition. On the one hand, the decline in fertility and mortality may give more freedom to women to participate in education and employment, as they expend less time in childbearing and childrearing. The extent to which women may benefit from these opportunities mainly depends on the different gender roles in non-market work. On the other hand, technological development in the workplace coincides with increasing educational attainment.

These trends are closely related, but the causality between them remains an open question in the economic literature. Lack of data availability, especially in the longitudinal dimension, is often an important constraint in modelling the effects of ageing in general equilibrium models.

The initial OLG model (Samuelson, 1958; Diamond, 1965) was designed mainly to introduce endogenous savings along the life cycle in the presence of changes in population structure. Since then, a growing literature has focused on different aspects of socioeconomic changes occurring during the demographic transition. Together with the theoretical literature considering different aspects of the complex relationships among related variables (fertility, labour supply, human capital investments, productivity growth, etc.), a more applied literature in large scale applied general equilibrium models (OLG-CGE) has developed. Starting with the seminal work by Auerbach and Kotlikoff (1987), these models allow for a detailed analysis of the effects of ageing on public expenditures and revenues. Again, these models usually focus on particular aspects, such as the feasibility of the pension system, and endogenise only some of the variables involved in the AGENTA project.[1]

Our modelling strategy aims at accounting coherently for the socioeconomic processes mentioned above. At the same time, we will emphasize applicability, working towards producing robust simulation results for the issues that are pushing the policy agenda.

NTA-based OLG-CGE modelling began with Sanchez-Romero *et al*. (2013a, 2013b). The first model focuses on the effect of transfers on savings in the face of population aging, while the second model analyses the role of recent pension reforms for the development of the social security system and economic growth in Austria. Both models are simulated backward and forward in time in order to study the development of transfers. Using NTTA data, we will extend these models by incorporating non market private transfers. Starting from the application in Apps and Rees (2001), we will also be able to examine and include

[1] In this regard, one of the issues that has received special attention is the analysis of the effects of the pension system on capital deepening as the baby boomers retire (Börsch-Supan et al. 2002 and 2006).

household production. Moreover, we will extend the above OLG-CGE models in several other directions. First, we will endogenise educational investments (schooling and on-the-job training) and other expenditures on children. Second, we will extend the labour supply decision, already endogenised in Sanchez-Romero et al. (2013b), based on Keuschnigg, Keuschnigg, and Jaag (2011). The latter article, which models the labour supply at various margins, includes job search and on-the-job training, which will probably need to be simplified in order to focus on other key variables. We will use the results of retrospective analysis together with the work on the differential role of mortality on education by Cervellati and Sunde (2013) and on retirement by d'Albis (2012) in order to define how education and the retirement process are modelled along the demographic transition. The next section details the variables needed in order to calibrate the model.

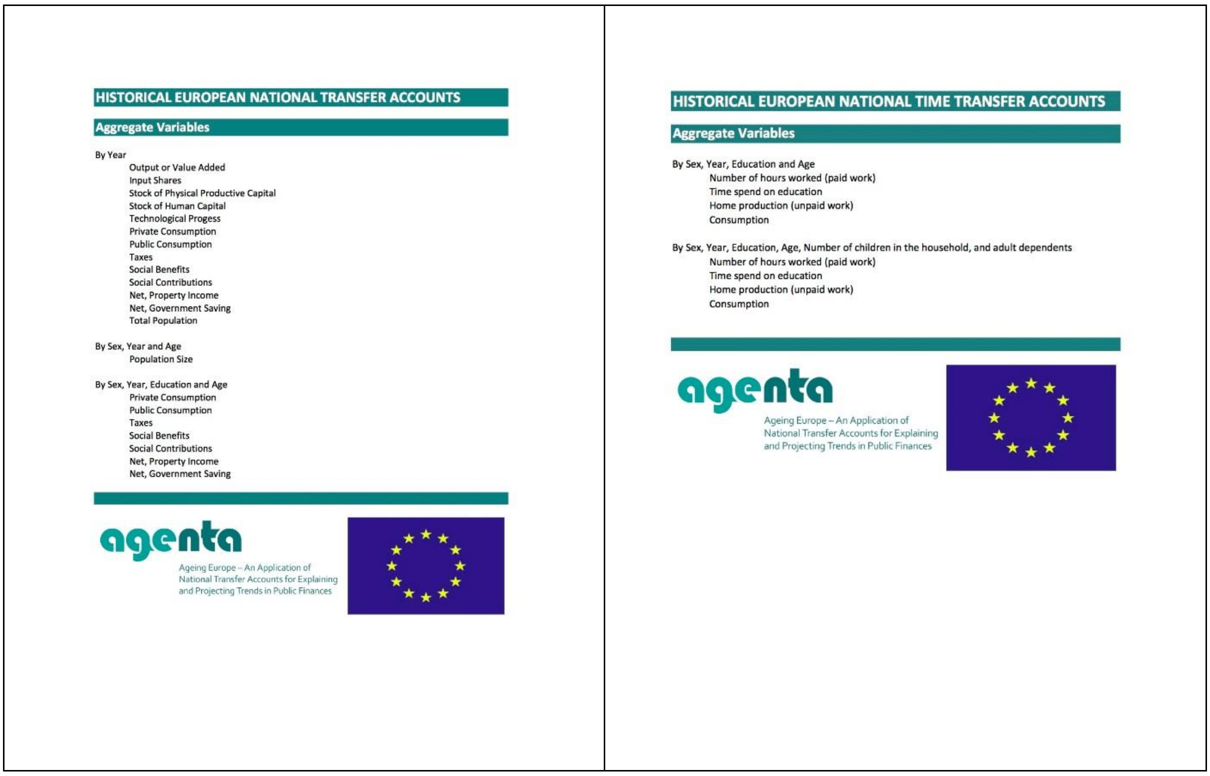
3 Variables required to derive the retrospective calibration process

One of the main features of the model designed is the retrospective calibration process. This entails, in contrast with many models in the literature, starting from a stable population, which improves the convergence and calibration of the model. However, this calibration process implies quite intensive data requirements. On the one hand, it is necessary to reconstruct the past evolution of population and link this evolution to a hypothetical initial stable population. Appendix A details the methods used in that process both for the case of one sex and two sex models. On the other hand, once population is recovered, the economic variables are collected in order to track the evolution of some targeted variables. This process includes the calculation of the labour augmenting technological progress, as explained in Appendix B, the collection of national accounts (NA) as macro controls and the use of NTA and NTTA profiles.

The variables required are listed below. The demographic variables, listed in subsection A, are necessary to reconstruct the past population and to project it to the future, both in the case of the one sex model and the two sexes model. The variables listed in subsections B.1 and B.2 include the main components of GDP using the production, the income and the expenditure approach. Those referring to the measurement of effective labour need to be collected by educational attainment at least for the recent past. Finally, subsections B.3 and B.4 are basically the NTA and NTTA profiles respectively. Ideally this should be available for several years, but at least a recent year which is neutral in terms of the business cycle is necessary. In this line, the retrospective macroeconomic analysis done in WP4 will turn out crucial to isolate the effect of the business cycle in the NTA profiles.

3.1 Variable list

The variables requested from each country will be stored in two spreadsheets (see the front page of each spreadsheet in Figure 1). These two spreadsheets are built to create a database for the countries involved in the AGENTA project that will help to calibrate the OLG-CGE model and to easily compare the different transfer systems among countries over time distinguishing by age, sex, and educational attainment.


HISTORICAL EUROPEAN NATIONAL TRANSFER ACCOUNTS

Aggregate Variables

By Year
- Output or Value Added
- Input Shares
- Stock of Physical Productive Capital
- Stock of Human Capital
- Technological Progess
- Private Consumption
- Public Consumption
- Taxes
- Social Benefits
- Social Contributions
- Net, Property Income
- Net, Government Saving
- Total Population

By Sex, Year and Age
- Population Size

By Sex, Year, Education and Age
- Private Consumption
- Public Consumption
- Taxes
- Social Benefits
- Social Contributions
- Net, Property Income
- Net, Government Saving

Ageing Europe – An Application of National Transfer Accounts for Explaining and Projecting Trends in Public Finances

HISTORICAL EUROPEAN NATIONAL TIME TRANSFER ACCOUNTS

Aggregate Variables

By Sex, Year, Education and Age
- Number of hours worked (paid work)
- Time spend on education
- Home production (unpaid work)
- Consumption

By Sex, Year, Education, Age, Number of children in the household, and adult dependents
- Number of hours worked (paid work)
- Time spend on education
- Home production (unpaid work)
- Consumption

Ageing Europe – An Application of National Transfer Accounts for Explaining and Projecting Trends in Public Finances

Figure 1: Front page of Variable list NTA (left-hand side) and NTTA (right-hand side)

A Demographic variables

The information requested in this subsection aims at collecting sufficient demographic data for reconstructing the historical population of each country involved in the AGENTA project. Since in many European countries the first censuses started in the middle of the XIX century, our goal would be to properly estimate the population from year 1800 up to now. The following list of demographic variables should be reported by sex, when the information exists, as well as in total over both sexes.

a. Total population
b. Total number of births
c. Total number of deaths
d. Population distribution by age (censuses)

e. Age-specific mortality rates
f. Age-specific fertility rates
g. Age-specific migration rates
h. Crude death rate
i. Crude birth rate
j. Crude migration rate

The methodology that will be applied to reconstruct the population by one and two sexes is explained in Appendix A.

B Economic variables

The information collected in this subsection is divided into four groups. The first group (B.1) is comprised of variables that are necessary for calibrating the model and constructing the production input factors. The methodology we will use to construct the input factors is explained in Appendix B. The second and third group (B.2-3) list the main expenditure accounts (private and public) and government revenues. The information in these two groups should be reported not only by year, but also by sex, age, educational attainment and year. Hereinafter we use variables $\{i, x, h, t\}$ to denote sex, age, educational attainment and year, respectively. Educational attainment will be divided in four groups: no education, primary education, secondary education, and tertiary education, following the standard ISCED levels classification. The last group of variables (B.4) contains the information for modelling household production.

Provided the information, our calibration strategy will be to replicate: 1) the evolution of the main macroeconomic time series contained in the list of variables in B.1, and 2) the NTA/NTTA profiles reported in B.2-B.4.

B.1 Output or value added (Y_t): (GDP minus net indirect taxes)

a. Human capital stock (H_t)

- $Edu^{i}_{x,h,t}$: Share of people within each educational attainment group by sex, age and year

- $y^i_{x,h,t}$: Labour income by sex, age, education and year
- $e^i_{x,h,t}$: Employment rates by sex, age, education and year
- $\ell^i_{x,h,t}$: Number of hours worked by sex, age, education and year
- Labour force participation rates by sex, age, education and year (to be used for calculations of the effective retirement age)

b. Physical capital stock (K_t)

- Investment by category (Gross fixed capital formation, I_t)
- Consumption of fixed capital by category ($\delta_t K_t$)
- 10-year government bond

c. Input shares

- Compensation of employees ($w_t H_t$)
- Gross operating surplus ($(r_t + \delta_t)K_t$)

d. Technological Progress (A_t)

Variables $\{\delta_t, r_t, w_t\}$ stand for the physical capital depreciation rate in year t, the real interest rate in year t, and the wage rate per unit of effective hour worked in year t. The information requested should be expressed in real terms; otherwise the resulting labour-augmenting technological progress will be distorted by changes in prices.

Educational attainment information will be withdrawn from existing databases such as Lutz *et al.* (2014), Barro and Lee (2013), Loichinger (2012), Samir *et al.* (2010) and Lutz *et al*. (2007).

B.2 Expenditure accounts

a. Consumption

  i. Private consumption

1. Health
2. Education
3. Household imputed rent
4. Others

ii. Public consumption / Government final consumption

b. Investment (I_t)
c. Net exports (XN_t)

Note that (b) should coincide with the gross fixed capital formation. A detailed list of public expenditures as well as public revenues is specified in the next list of variables.

B.3 General Government – public age reallocations

a. Expenditures
    i. Public transfers, in-kind
        1. Health
        2. Education
        3. Long-term care
        4. Other in-kind
    ii. Public transfers, cash
        1. Pensions and social protection
            a. Retirement
            b. Disability
            c. Survivors
            d. Maternity
            e. Other pensions
        2. Unemployment benefits
        3. Other transfers, in cash
b. Revenues
    i. Taxes
        1. Taxes on income, profits and capital gains
        2. Taxes on payroll and workforce

3. Taxes on property
4. Taxes on goods and services (consumption taxes)
5. Taxes on international trade and transactions
6. Other taxes

ii. Social contributions

iii. Grants

c. Public interest, net
d. Property income, net
e. Saving, net

Moreover, it will be important to track public assets/debt when implementing the fiscal model. Notice these data can be collected from the International Monetary Fund (IMF)-Fiscal Monitor, however it should be validated with Eurostat data.

B.4 Time-use

The information requested in this subsection aims at understanding the evolution of time devoted to market and home production. All these variables should be reported by age, sex, education (of household head) {No education, Primary, Secondary, Tertiary}, number of children currently in the household {0,1,2,3+} and number of adult dependents:

a. Number of hours worked (paid work)
b. Time spent on education
    a. Formal
    b. Informal
c. Home production (unpaid work)
    a. Childcare (for children)
    b. Adult care (for dependent adults)
d. Consumption

Note: It should also be controlled by the number of offspring since it has a pronounced effect on the time transfers for women.

B.5 Other variables

a. Age of entry in the labour market over time (if possible by sex)
b. Age at exit of the labour market over time (if possible by sex): The results of the WP3 will be crucial to design the way the retirement decision is modelled.
c. Main parametric components of the pension system over time.

Whenever NTA/NTTA data will not be available, we will solve the inconsistencies between in and out transfer flows for both public and private sectors by using various scenarios. Possible scenarios might be that we adjust either inflows at 100%, or outflows at 100%, or 50-50.

Appendix A. Population reconstruction and projections

There are two fundamental agents in our analysis: individuals and households. To properly understand how these two agents interact over time and by age, among them, and with markets it is necessary to comprehend their demographic characteristics. Unfortunately, existing demographic databases do not provide information at the individual level consistent with that at the aggregate level. Frequent sources of discrepancy between information at the individual and at the population level are due to migration flows, the collection of vital statistics using alternative sources, the timing of collection of vital information, short time coverage, and age-grouping among many others. For this reason, in Working Package 5 existing demographic methods will be implemented to reconstruct historical populations and create in a consistent manner individual and household demographic information. Thus, unlike many OLG-CGE models, the demographic structure of the household will change over time due to mortality and fertility, although, for tractability reasons, they will only be comprised of parents and dependent children. This is an important step forward to build a large demographic dataset consistent with existing OLG economic models.

Depending upon the economic model to be used in each simulation we split our demographic reconstruction in two models. In Appendix A.1, we explain the methodology to reconstruct the population assuming a sexless world in which adults give birth to children hermaphroditically. Actually, this is the standard demographic assumption used in most OLG-CGE models. In Appendix A.2, we develop the methodology to realistically reconstruct the population considering that individuals differ not only by age but also by sex.

Appendix A. 1. One sex model

Let N_t denote the total population size in year t and $N_{x,t}$ be the size of the population at age x in year t. We initially assume an open population whose law of motion is

$$N_{t+1} = N_t + B_t - D_t + M_t. \tag{1}$$

Eq (1) implies that population at time $t+1$ is equal to the population in year t plus the total number of births in year t, denoted by B_t, less the total number of deaths during the year, or D_t, plus the total net migration flow in year t, or M_t. The dynamics of the population can be written in matrix notation using a Leslie matrix (Leslie,1945; Preston et al.,2002)

$$N(t+1) = L(t)N(t) + M(t) \tag{2}$$

where $L(t)$ satisfies

$$\begin{aligned} L_{1,x}(t) &= \frac{L_{0,t}}{2l_{0,t}}\left(f_{x,t} + f_{x+1,t}\frac{L_{x+1,t}}{L_{x,t}}\right)f_{fab} \\ L_{x+1,x}(t) &= \frac{L_{x+1,t}}{L_{x,t}}, \end{aligned} \tag{3a}$$

and $M(t)_{\Omega-1\times1}$ is the net migration vector in which

$$M_{x,t} = m_t m_x \text{ for } x \in \{1,\dots,\Omega-1\} \text{ at time } t, \tag{3b}$$

where $L_{x,t} = \frac{l_{x,t}+l_{x+1,t}}{2}$ is the person years lived by the cohort between ages x and $x+1$ in period t, $f_{x,t}$ is the age-specific fertility rate at age x in year t, f_{fab} is the fraction of females at birth (we assume $f_{fab} = 0.4886$, which is the standard value in the demographic literature), m_x is the age pattern of net migration, which is assumed to be invariant over time and its sum across age equals one, and m_t is the crude net migration rate in year t.

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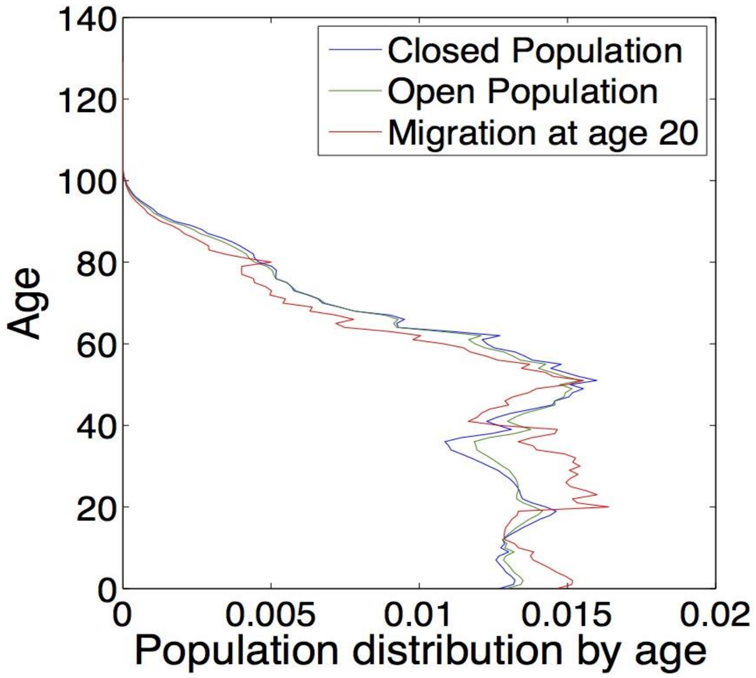

Figure 2. Population distribution by age according to different migration assumptions, USA 2000.

Source: Authors calculations based on US-CPS data.

Notes: Closed population line (blue solid line) represents the population distribution by age under the assumption that there were no migration flows from year 1850 up to now. Open population line (green solid line) depicts the current population distribution by age in year 2000. Migration at age 20 line (red solid line) represents the population distribution by age that would result if migrants are assumed to enter in the population at age 20 from year 1850 up to now.

Since the OLG-CGE economic model is not well implemented for studying the economic effect of migration, we initially remove migrants and analyse the model under a closed population structure. Notice that that, although this method does not take into account migrants, it is better suited for analysing the effect of changes in the population age structure on economic growth than a standard model that considers migration. This is because under a closed population (cf. green line and blue line in Figure 2) the relative size of each cohort with respect to the total is closer to the current population structure than under the assumption that all migrants enter at the age of 20, as it is frequently assumed (cf. red line and blue line in Figure 2).[2]

Later on, and depending upon data availability, we might use the open population estimates, obtained from the population reconstruction, to analyse the economic impact of migration.

Historical component. To reconstruct the population we first collect historical demographic information. The main variables are: total population, total births, total deaths, population distribution by age, age-specific fertility rates and age-specific mortality rates. In order to avoid age heaping problems, we pool age-groups and divide deaths across age groups by the associated population at risk. From this information we use splines to construct Life Tables by single-years of age and more importantly person-years lived at age x (L_x) and person-years lived above age x (T_x), which are a posteriori used to get rid of the observed age-heaping. At very old ages we fit the mortality rate using the model proposed by Perks (1932) and Kannisto (1992):

$$\mu(x) = \frac{\alpha\beta^x}{1+c^{-1}\alpha\beta^x}, \tag{4}$$

which guarantees that death rates increase at a diminishing rate, with an asymptote in c as x ↑∞.

For the initial population, we make use of Lotka's stable population theory (Lotka, 1998). Realize that this is an assumption necessary to guarantee the economy starts from an initial steady-state. Thus, our initial population at age x is given by

$$N_{x,t} = N_{0,t}e^{-rx}L_{x,t}, \text{ for } t < t_0, \tag{5}$$

where r is the annual population growth rate and t_0 is the first year with a non-stable population.

[2] Exceptionally, an open population could be used if migrants had exactly the same demographic and socio-economic characteristics as those of nationals.

To reconstruct the actual population of each country, we combine two demographic methods widely used in population reconstruction: Inverse Projection (IP) and Generalized Inverse Projection (GIP) (Lee, 1985; Oeppen, 1993). We use a simplified version of a GIP model that matches the specific characteristics of the demographic data gathered for our purpose. The objective function is:

$$\min_{\{\alpha_t^i, m_t, \kappa_t\}} \sum_{t\in\mathcal{D}}\left(\frac{D_t-\widehat{D}_t}{D_t}\right)^2+\sum_{t\in\mathcal{B}}\left(\frac{B_t-\widehat{B}_t}{B_t}\right)^2+\sum_{t\in\mathcal{N}}\left(\frac{N_t-\widehat{N}_t}{N_t}\right)^2+\sum_{t\in\mathcal{M}}\left(\frac{M_t-\widehat{M}_t}{M_t}\right)^2+ \sum_{t\in\mathcal{E}}\left(\frac{e_{0,t}-\hat{e}_{0,t}}{e_{0,t}}\right)^2+\sum_{t\in\mathfrak{T}}\left(\frac{tfr_t-\widehat{tfr}_t}{tfr_t}\right)^2+\sum_{t\in\mathbb{C}}\sum_{a=2}^{15}\left(\frac{{}_4N_{5a,t}-{}_4\widehat{N}_{5a,t}}{N_t}\right)^2+ \sum_{t\in\mathbb{D}}\sum_{a=2}^{15}\left(\frac{{}_4D_{5a,t}-{}_4\widehat{D}_{5a,t}}{D_t}\right)^2+\sum_{t\in\mathbb{T}}\sum_i\left(\alpha_{t+1}^i-\alpha_t^i\right)^2+\left(\kappa_{t+1}-\kappa_t\right)^2+ \left(m_{t+1}-m_t\right)^2, \quad (6)$$

subject to equations (2)-(3) and to

$$f_{x,t}=\sum_{j=1}^3\alpha_t^j f_x^{(j)}, \sum_{i=1}^3\alpha_t^j\le 1, \alpha_t^j\ge 0, \quad (7)$$

$$\mu_{x,t}=e^{a_x+\kappa_t b_x}, \quad (8)$$

$$\underline{m}\le m_t\le\overline{m}, \quad (9)$$

where variables with the symbol ($\hat{}$) are the estimated values. Moreover, $\left\{\{\alpha_t^j\}_{j=1}^3, \kappa_t, m_t\right\}$ is the set of parameters for fertility, mortality and migration in year t, respectively, $\left\{f_x^{(j)}\right\}_{j=1}^3$ are actual age-specific fertility rates for some specific years, $\{a_x, b_x\}$ are the splines associated to the Lee-Carter model (Lee and Carter, 1992), tfr stands for total fertility rate, e_0 denotes life expectancy at birth and $\{\mathcal{D},\mathcal{B},\mathcal{N},\mathcal{M},\mathcal{E},\mathcal{L},\mathbb{C},\mathbb{D},\mathbb{T}\}$ is the set of available years for which there exists information on total deaths, total births, total population, total flow of net migration, life expectancy, total fertility rates, censuses, and death rates, respectively. To avoid incorrect population data collected at old ages, notice we only use the population census by five-years age groups from age 5 up to age 80. Crude net migration rates are obtained using inverse population projection and are exogenously introduced in the GIP model.

Forecast. From the last available demographic data to the future, our population projections will be based either on Eurostat assumptions or on UN Population Division assumptions depending upon data availability.

Results. The information obtained with the population reconstruction will mainly be used to derive households by age of the household head, to specify the mortality and fertility risks associated to each individual across the lifecycle, and to determine the number and age of probable surviving siblings and parents that are consistent with the existing population structure.

Regarding the first goal, we will follow the framework set out by Tobin (1967), Lee (1980), Mason (2002) and more recently by Sanchez-Romero (2013) in developing the household model. The household framework, even considering one representative adult consumer, has been found crucial for explaining the excess sensitivity of consumption to income (Blundell et al., 1994; Attanasio and Browning, 1995; Attanasio and Weber, 1995 and 2010). Thus, models that take into account the cost of children better fit the savings pattern than those that do not (Browning and Ejrnaes, 2009).

In this framework surviving adults are counted as one unit, while children are expressed in equivalent adult units, denoted by θ, that varies by age according to

$$\theta_x = \begin{cases} 0.4 & \text{If } 0 \leq x \leq 4 \\ 0.4 + (0.3/7)(x-4) & \text{If } 4 < x < 18. \\ 1 & \text{If } x \geq 18 \end{cases} \tag{10}$$

The household size with head age x born in year t^c is

$$\eta_{x,t^c} = 1 + \sum_{a=1}^{x-1} \frac{l_{a,t^c}}{l_{x,t^c}} f_{a,t^c} l_{x-a,t^c+a} \theta_{x-a}, \tag{11}$$

where t^c is the year of birth. Importantly, for consistency reasons, both variables f and l are calculated using the Leslie matrix (see Eq. 3) rather than period life tables. From (11), the total consumption of a household whose head is age x and was born in year t^c is given by

$$C_{x,t^c} = c_{x,t^c}\eta_{x,t^c}, \quad (12)$$

where c_{x,t^c} is the consumption of the household head. Similarly, the total consumption in year t is the sum of household consumptions across age of household heads

$$C_t = \sum_{x=z}^{\Omega-1} C_{x,t-x} N_{x,t}, \quad (13)$$

where z is the age at which an adult forms a household. Note that our methodology incorporates in the economic model not only adults but also children, allowing to introduce the two dependent stages of the lifecycle.

Appendix A. 2. Two sexes model

Let N_t^m, N_t^f be the total male and female populations in year t, respectively, and $N_{x,t}^m$, $N_{x,t}^f$ be the size of the male and the female population at age x in year t, respectively. Like with the one-sex model, we initially assume an open population whose law of motion is given by the following three equations

$$\begin{aligned} N_t &= N_t^m + N_t^f, \\ N_{t+1}^m &= N_t^m + B_t^m - D_t^m + M_t^m, \quad (14) \\ N_{t+1}^f &= N_t^f + B_t^f - D_t^f + M_t^f, \end{aligned}$$

The total population in year t, denoted by N_t, is given by the sum of the male and female populations in year t. Note that superscripts f and m denote the sex. The dynamics of the female and male populations can be written in matrix notation as follows

$$\begin{aligned} N^f(t+1) &= L^f(t)N^f(t) + M^f(t), \\ N^m(t+1) &= L^m(t)N^m(t) + M^m(t), \end{aligned} \quad (15)$$

where $L^f(t)$ and $L^m(t)$ are matrices of zeros except for

$$L_{1,x}^{f}(t)=\frac{L_{0,t}^{f}}{2l_{0,t}^{f}}\left(f_{x,t}+f_{x+1,t}\frac{L_{x+1,t}^{f}}{L_{x,t}^{f}}\right)f_{fab},$$

$$L_{x+1,x}^{f}(t)=\frac{L_{x+1,t}^{f}}{L_{x,t}^{f}}, \tag{16a}$$

$$L_{x+1,x}^{m}(t)=\frac{L_{x+1,t}^{m}}{L_{x,t}^{m}}, \text{ for all } x\in\{1,\cdots,\Omega-1\},$$

and

$$N_{1,t}^{m}=L_{1,\cdot}^{f}(t)N^{f}(t)\frac{L_{0,t}^{m}}{L_{0,t}^{f}}\frac{l_{0,t}^{f}}{l_{0,t}^{m}}\frac{1-f_{fab}}{f_{fab}},$$

$$M_{x,t}^{f}=m_{t}^{f}m_{x}^{f}, \tag{16b}$$

$$M_{x,t}^{m}=m_{t}^{m}m_{x}^{m} \text{ for } x\in\{1,\dots,\Omega-1\} \text{ at time } t,$$

where $L_{x,t}^{i}=\frac{l_{x,t}^{i}+l_{x+1,t}^{i}}{2}$ is the sex-specific, $i\in\{f,m\}$, person years lived by the cohort between ages x and $x+1$ in period t, $f_{x,t}$ is the age-specific fertility rate at age x in year t, f_{fab} is the fraction of females at birth (we assume $f_{fab}=0.4886$, which is the standard value in the demographic literature), m_{x}^{i} is the sex-specific age pattern of net migration, which is also assumed to be invariant over time and its sum across age equals one, and m_{t}^{i} is the sex-specific crude net migration rate in year t.

Similar to the one-sex model we combine the Inverse Projection (IP) and the Generalized Inverse Projection (GIP) (Lee, 1985; Oeppen, 1993) in order to reconstruct the female and male populations.

$$\begin{aligned}
&\min_{\{\alpha_t^j,m_t^i,\kappa_t^i\}}\sum_i\sum_{t\in\mathcal{D}^i}\left(\frac{D_t^i-\widehat{D}_t^i}{D_t^i}\right)^2+\sum_i\sum_{t\in\mathcal{B}^i}\left(\frac{B_t^i-\widehat{B}_t^i}{B_t^i}\right)^2+\sum_i\sum_{t\in\mathcal{N}^i}\left(\frac{N_t^i-\widehat{N}_t^i}{N_t^i}\right)^2+\\
&\sum_i\sum_{t\in M^i}\left(\frac{M_t^i-\widehat{M}_t^i}{M_t^i}\right)^2+\sum_i\sum_{t\in\mathcal{E}^i}\left(\frac{e_{0,t}^i-\hat{e}_{0,t}^i}{e_{0,t}^i}\right)^2+\sum_{t\in\mathfrak{T}}\left(\frac{tfr_t-\widehat{tfr}_t}{tfr_t}\right)^2+\\
&\sum_i\sum_{t\in\mathbb{C}^i}\sum_{a=2}^{15}\left(\frac{{}_4N_{5a,t}^i-{}_4\widehat{N}_{5a,t}^i}{N_t^i}\right)^2+\sum_i\sum_{t\in\mathbb{D}^i}\sum_{a=2}^{15}\left(\frac{{}_4D_{5a,t}^i-{}_4\widehat{D}_{5a,t}^i}{D_t^i}\right)^2+\\
&\sum_{t\in\mathbb{T}}\left(\sum_j\left(\alpha_{t+1}^j-\alpha_t^j\right)^2+\sum_i\left(\left(m_{t+1}^i-m_t^i\right)^2+\left(\kappa_{t+1}^i-\kappa_t^i\right)^2\right)\right),
\end{aligned} \tag{17}$$

subject to equations (15)-(16) and to

$$f_{x,t} = \sum_{j=1}^{3} \alpha_t^j f_x^{(j)}, \sum_{j=1}^{3} \alpha_t^j \leq 1, \alpha_t^j \geq 0\,, \quad (18)$$

$$\mu_{x,t}^i = e^{a_x^i + \kappa_t^i b_x^i}, \quad (19)$$

$$\underline{m}^i \leq m_t^i \leq \overline{m}^i, \text{ for } i \in \{f, m\}. \quad (20)$$

where $\left\{\{\alpha_t^j\}_{j=1}^3, \{\kappa_t^i, m_t^i\}_{i\in\{f,m\}}\right\}$ is the set of parameters for fertility, mortality and migration in year t, respectively, $\left\{f_x^{(j)}\right\}_{j=1}^3$ are actual age-specific fertility rates for some specific years, $\{a_x^i, b_x^i\}$ are the splines associated to the Lee-Carter model for sex $i \in \{f, m\}$ (Lee and Carter, 1992), tfr stands for total fertility rate, e_0^i denotes the life expectancy at birth of sex i, and $\{\mathcal{D}^i, \mathcal{B}^i, \mathcal{N}^i, \mathcal{M}^i, \mathcal{E}^i, \mathcal{L}^i, \mathbb{C}^i, \mathbb{D}^i, \mathbb{T}\}$ is the set of available years for which there exists information for sex $i \in \{f, m\}$ on total deaths, total births, total population, total flow of net migration, life expectancy, total fertility rates, censuses, and death rates, respectively. Moreover, to avoid incorrect population data collected at old ages, we use the population census by five-years age groups from age 5 up to age 80.

Forecast. Future population projections follow Eurostat assumptions.

Results. The population reconstruction will be used to derive households, to specify the mortality and fertility risks associated to each individual and sex across the lifecycle, and to determine the number and age of probable surviving siblings and parents that are consistent with the existing population structure.

In this model we move away from a single headed household to a household consisting of a family with two possible heads. The economic literature dealing with the family life cycle is not very extensive although it has a very strong theoretical foundation (Apps and Rees, 2001, 2003, 2005, 2009). Since the possible combination of demographic-socio-economic events (e.g. age at partnering, childrearing cost distribution, among many others) substantially increases when two individuals interplay, for simplicity, we introduce the following assumptions: 1) agents pair with individuals with the same age from the opposite sex; 2) adults, living in the same household, allocate the cost of rearing children equally, according to their consumption power; 3) similar to the one-sex model, orphans are assumed to be raised by surviving individuals with similar demographic characteristics as their deceased parents; and 4) the consumption needs of children are assumed to change only by age and not by sex (see Eq. 10). Therefore, the number of adult consumers supported by an adult of age x born in year t^c, regardless the sex, is

$$\eta_{x,t^c} = 1 + \frac{\sum_{a=1}^{x-1} N^f_{a,t^c} f_{a,t^c} \theta_{x-a} \left[l^f_{x-a,t^c+a} f_{fab} + l^m_{x-a,t^c+a} (1 - f_{fab}) \right]}{N^m_{x,t^c} + N^f_{x,t^c}}, \qquad (21)$$

where t^c is the year of birth, θ_x is the adult equivalent consumption unit at age x, f is the fertility, and l is the survival probability. Comparing Eq (21) to (11) we find two important features. First, if both sexes have the same survival probability, the number of adult consumers supported by an adult of age x born in year t^c coincides with Eq. (11). Hence, Eq. (21) is a generalization of Eq. (11). Second, if all males die out at age x, the cost of rearing children is completely borne by surviving females. This feature cannot be obtained in a one-sex model. From (21), the total expenditure of an adult at age x born in year t^c and sex i is

$$C^i_{x,t^c} = c^i_{x,t^c} \eta_{x,t^c}, \text{ for } i \in \{f, m\} \qquad (22)$$

where c^i_{x,t^c} is the consumption of an adult of age x and sex i who was born in year t^c. Then, the total consumption in year t is given by

$$C_t = \sum_i \sum_{x=z}^{\Omega-1} C^i_{x,t-x} N^i_{x,t}, \qquad (23)$$

where z is the age at which an adult forms a household.

Appendix B. Economic information

In this section we deal with the calculation of the production input factors: the stock of physical capital and the stock of human capital. The determination of the input factors is necessary for a twofold motive. First, it is needed for the calibration of the model so as to the replicate the particular economy. Second, it is necessary in order to calculate the labour-augmenting technological progress.

Appendix B. 1. Input factors

To derive the labour-augmenting technological progress we make use of the standard technique used in the growth accounting literature (Bloom and Williamson, 1998; Kelley and Schmidt, 2005; Sanchez-Romero, 2013). In particular, using a Cobb-Douglas production function with constant returns to scale it can be shown that the logarithm of income per capita is

$$\log\frac{Y_t}{N_t} = \frac{\alpha_t}{1-\alpha_t}\log\frac{K_t}{Y_t} + \log A_t + \log\frac{H_t}{N_t}, \tag{24}$$

where Y_t is the value added product (at factor cost) in year t, N_t is the population size in year t, α_t is the capital share, or gross operating surplus divided by the value added in year t, K_t is the stock of physical productive capital in year t, A_t is the labour-augmenting technology in year t, and H_t is the stock of human capital in year t.

Stock of physical capital. In order to calculate the stock of physical capital we apply the perpetual inventory approach to gross fixed capital formation by category (i.e. construction, transport equipment, machinery and equipment, and intangible fixed assets). Following Hulten and Wykoff (1981) we use to each category the following the depreciation rates: 2.1, 18.2, 13.8, and 15%, respectively. The initial capital stock is computed as $K_0^i = I_0^i/(\delta_i + g_i)$, where I_0^i is the initial investment in category i, δ_i is the depreciation rate of category i, and g_i is the growth rate of investment in the first 10 years in category i. The perpetual inventory approach formula is shown in Eq. (25). In case of not having information by category, we instead calculate the capital stock using the aggregate gross fixed capital formation and the consumption of fixed capital.

$$K_{t+1}^i = K_0^i + \sum_{s=0}^{t} I_s^i - \delta_s^i K_s^i, \tag{25}$$

$$K_t = \sum_i K_t^i. \tag{26}$$

Since the initial capital stock is somehow arbitrary, the standard approach in the literature is to discard the first 15 years of the capital stock obtained using Eq. (25). Therefore, although our time series span for T years, we will end up having T-15 years of data.

Stock of human capital. It is important to keep in mind that the total units of human capital not only depends on the number of workers but also depends on the labour supply and the productivity of each worker. Thus,

$$H_t = \sum_{i\in\{f,m\}} \sum_x N_{x,t}^i \sum_{h\in\mathcal{H}} e_{x,h,t}^i \ell_{x,h,t}^i \epsilon_{x,h}^i Edu_{x,h,t}^i\,, \tag{27}$$

where the set $\mathcal{H}$={No education, primary, secondary, tertiary} represents the educational attainment groups.[3] The first term in Eq. (27), or $N_{x,t}^i$, is the population size of sex i at age x in year t, the second term is the employment rate of sex i at age x and educational attainment h in year t ($e_{x,h,t}^i$), the third term ($\ell_{x,h,t}^i$) is the intensive labor supply or hours worked by sex i at age x and educational attainment h in year t, the fourth term ($\epsilon_{x,h}^i$) is the productivity of sex i at age x and educational attainment h, and the last term ($Edu_{x,h,t}^i$) is the share of people of sex i at age x with education h in year t. Note that $\epsilon_{x,h}^i$ does not change over time. This is because the change in workers' productivity over time should be captured by the labour-augmenting technological progress and not by the stock of human capital. Thus, in order to derive $\epsilon_{x,h}^i$ we will use econometric techniques to try to decompose the labour income profiles for each educational group (conditional on being employed) by age, cohort, and time effects.

[3] Notice that by distinguishing different educational groups we can better assess the evolution of the stock of human capital devoted to market work as suggested by Schultz (1961), since the productivity of each worker, the employment rate, and hours worked are highly correlated to the educational attainment.

Labour-augmenting technological progress. Differentiating Eq. (24) with respect to time and rearranging, we obtain the labour-augmenting technological progress growth rate in year t

$$g_t^A = g_t^{Y/N} - (g_t^H - g_t^N) - \frac{\alpha_{t+1}-\alpha_t}{(1-\alpha_t)^2}\log\frac{K_t}{Y_t} - \frac{\alpha_t}{1-\alpha_t} g_t^{K/Y}, \qquad (28)$$

where g_t^X denotes the growth rate of variable X from year t to $t+1$. In simple words, g_t^A is the exogenous growth rate of the economy over time.

Correspondingly, using Eq. (28) we can derive the contribution to per capita output growth in different components:

$$g_t^{Y/N} = (g_t^H - g_t^N) + \left(\frac{\alpha_{t+1}-\alpha_t}{(1-\alpha_t)^2}\log\frac{K_t}{Y_t} + \frac{\alpha_t}{1-\alpha_t} g_t^{K/Y} + g_t^A\right). \qquad (29)$$

The first parenthesis on the right-hand side of Eq. (29) is known as the "Translation Component" (Bloom and Williamson, 1998; Kelley and Smith, 2005). The second term on the right-hand side is known as the "Productivity Component". Since g_t^A is exogenous to the model, the goal with the requested variable list is to minimize the "unexplained" per capita output growth component. This is possible because as the number of variables explaining the stock of human capital increases, the higher is the contribution of the translation component to per capita output growth.

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References

Apps, P., Rees, R. (2001). Household production, full consumption and the costs of children. *Labour Economics* 8(6), 621-648.

Apps, P., Rees, R. (2003). *Life cycle time allocation and saving in an imperfect capital market*. NBER Summer Institute Session on Aggregate Implications of Microeconomic Consumption Behavior, Boston, 21-25 July.

Apps, P., Rees, R. (2005). Gender, time use, and public policy over the life cycle. *Oxford Review of Economic Policy 21*(3), 439-461.

Apps, P., Rees, R. (2009). *Public economics and the household*. Cambridge University Press.

Attanasio, O. P., Browning, M. (1995). Consumption over the life cycle and over the business cycle. *American Economic Review* 85(5), 1118-1137.

Attanasio, O. P., Weber, G. (1995). Is consumption growth consistent with intertemporal optimization? Evidence from the Consumer Expenditure Survey. *Journal of Political Economy 103*(6), 1121-1157.

Attanasio, O.P., Weber, G. (2010). Consumption and saving: models of intertemporal allocation and their implications for public policy. *Journal of Economic Literature* 48(3), 693–751.

Auerbach, A. J., Kotlikoff, L.J. (1987): *Dynamic fiscal policy*. Cambridge: Cambridge University Press.

Barro, R., Lee, J.-W. (2013). A new data set of educational attainment in the World, 1950-2010. *Journal of Development Economics* 104, 184-198.

Bloom, D.E., Williamson, J.G. (1998). Demographic transitions and economic miracles in emerging Asia. *The World Bank Economic Review 12*(3), 419-455.

Blundell, R., Browning, M., Meghir, C. (1994). Consumer demand and the life-cycle allocation of household expenditures. *The Review of Economic Studies 61*(1), 57-80.

Börsch-Supan, A., Heiss, F., Ludwig, A., Winter, J. (2002). Pension reform, capital markets and the rate of return. *German Economic Review* 4 (2), 151–181.

Börsch-Supan, A., Ludwig, A., Winter, J. (2006). Ageing, pension reform and capital flows: A multi-country simulation model. *Economica*, London School of Economics and Political Sciences 73 (292), 625–658.

Browning, M., Ejrnaes, M. (2009). Consumption and children. *The Review of Economics and Statistics* 91 (1), 93–111.

Cervellati, M. Sunde, U. (2013). Life expectancy, schooling, and lifetime labor supply: Theory and evidence revisited. *Econometrica* 81 (5), 2055-2086.

d'Albis, H., Lau, P., Sanchez-Romero, M. (2012). Mortality transition and differential incentives for early retirement. *Journal of Economic Theory* 147 (1), 261-283.

Diamond, P.A. (1965). National debt in a neoclassical growth model. *American Economic Review* 55, 1126-1150.

Hulten, C.R., Wykoff, F.C. (1981). The measurement of economic depreciation. In Hulten, C.R. (ed) Depreciation, inflation, and the taxation of income from capital. The Urban University Press, Washington DC, 81-125.

Kannisto, V. (2001). Mode and dispersion of the length of life. *Population: An English Selection*, 159-171.

Kelley, A.C., Schmidt, R.M. (2005). Evolution of recent economic-demographic modeling: A synthesis. *Journal of Population Economics 18*(2), 275-300.

Keuschnigg, C., Keuschnigg, M., Jaag, C. (2011). Aging and the financing of social security in Switzerland. *Swiss Journal of Economic and Statistics* 147(2), 181-231.

Lee, R.D. (1980). Age structure intergenerational transfers and economic growth: an overview. *Revue économique*, 1129-1156.

Lee, R.D. (1985). Inverse projection and back projection: A critical appraisal, and comparative results for England, 1539 to 1871. *Population Studies 39*(2), 233-248.

Lee, R. D., and Carter, L. R. (1992). Modeling and forecasting US mortality. *Journal of the American statistical association*, *87*(419), 659-671.

Leslie, P.H. (1948). Some further notes on the use of matrices in population mathematics. *Biometrika*, 213-245.

Loichinger, E. (2012). Europe's future human capital: Population projections by age, sex, education, and labor force participation. Dissertation. Vienna, University of Economics and Business.

Lotka, A.J. (Ed.). (1998). *Analytical theory of biological populations*. Springer.

Lutz, W., Butz, W.P., Samir, K.C. (Eds) (2014). *World population and human capital in the twenty-first century*. Oxford University Press.

Lutz, W., Goujon, A., K.C., S and Sanderson, W. (2007). Reconstruction of population by age, sex and level of educational attainment of 120 countries for 1970-2000. *Vienna Yearbook of Population Research*. 2007, 193-235

Mason, A. (2002). Population and economic growth in East-Asia. In: Mason, A. (ed). *Population change and economic development in East Asia: challenges met, opportunities seized*. Stanford University Press, 1-30.

Oeppen, J. (1993). Back projection and inverse projection: members of a wider class of constrained projection models. *Population Studies 47*(2), 245-267.

Perks, W. (1932). On some experiments in the graduation of mortality statistics. *Journal of the Institute of Actuaries*, 12-57.

Preston, S. H., Heuveline, P., Guillot, M. (2001). Demography: measuring and modeling population processes. Blackwell Publishers, Oxford.

Samir, K.C., Barakat, B., Goujon, A., Skirbekk, V., Sanderson, W.C., Lutz, W. (2010). Projection of populations by level of educational attainment, age, and sex for 120 countries for 2005-2050. *Demographic Research* 22(15), 383-472.

Samuelson, P. (1958): An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of Political Economy* 66(6), 467-482.

Sánchez-Romero, M. (2013). The role of demography on per capita output growth and saving rates. *Journal of Population Economics* 26(4), 1347-1377. DOI 10.1007/s00148-012-0447-3

Sánchez-Romero, M., Patxot, C., Rentería, E., Souto, G. (2013a). On the effects of public and private transfers on capital accumulation: Some lessons from the NTA aggregates. *Journal of Population Economics* 26(4), 1409-1430. DOI 10.1007/s00148-012-0422-z.

Sánchez-Romero, M., Sambt, J., Prskawetz, A. (2013b). Quantifying the role of alternative pension reforms on the Austrian economy. *Labour Economics* 22(C), 94-114.

Shultz, T.W. (1961). Investment in human capital. *The American Economic Review* 51(1), 1-17.

Tobin, J. (1967). Life cycle saving and balanced growth. *Ten economic studies in the tradition of Irving Fisher. New York: Wiley*, 231-256.

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